A Theory of Power Wars*

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Abstract

This paper provides a theory of how war onset and war duration depend on the initial distribution of power when conflict triggers a reallocation of power but the loser is not eliminated. In the model, players take into account not only the expected consequences of war on the current distribution of resources, but also its expected consequences on the future distribution of military and political power. We highlight three main results: the key driver of war, in both the static and the dynamic game, is the *mismatch* between military and political power; dynamic incentives usually amplify static incentives, leading forward-looking players to be more aggressive; and a war is more likely to last for longer if political power is initially more unbalanced than military power and the politically under-represented player is militarily advantaged.

Keywords: Formal Model; International Relations; Causes of War; Dynamic Game; War Onset; War Duration; Balance of Power; Power Mismatch; Power Shift; Civil Wars; Inter-State Wars

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1 Introduction

The international relations debates on the importance of the balance of power for peace preservation are endless. At the same time, when scholars talk about balance of power versus preponderance of power, they always refer to a single measure of power, military power. Similarly, when the formal theory literature on the rationalist explanations for war points to asymmetric information about relative strength and lack of commitment in the future use of power as the main drivers of conflict, it mostly focuses on a one-dimensional notion of power. As pointed out by Filson and Werner (2002), the simultaneous consideration of multiple measures of relative power can uncover important elements to advance our understanding of conflict onset. This paper takes this insight seriously and provides a general, yet tractable, framework that delivers predictions about war outbreak and its dynamics, hence also allowing us to relate the findings to the literature on power shifts and power transition theory. We argue that characterizing the incentives to attack and its root causes— both before war breaks out and after a first war has occurred — is essential because all types of power can change as a consequence of conflict. Taking into account these novel dynamic incentives will clarify what we miss when neglecting the incentives coming from the present mismatch between relative military power and relative political power and from the expected consequences of war on the future distribution of powers.

While the advantages of balanced (military) power discussed in the literature are mostly coming from deterrence arguments (see Powell, 1996, 1999, and the early seminal work by Waltz, 1979), the importance of considering multiple dimensions of power was already a suggested necessity in another important strand of the international relations literature, namely, the so called "power transition theory." In a nutshell, according to this theory, there are two key variables that drive an interstate war: the relative military power (affecting feasibility) and the degree of dissatisfaction with the status quo of the international order (think of this as the motivation aspect for war). Hence, in contrast with the deterrence arguments highlighting the peace consequences of a balanced military power, this theory emphasizes that, even if two countries have approximately equal power, there can be war when there is asymmetry in dissatisfaction with the status quo. The arguments in the initial contributions to power transition theory were not formalized, and bargaining and costs were not considered, but the informal considerations on which the debate grew were inherently dynamic: Organski and Kugler (1980) stated that it is usually the weaker but grown-stronger state the most likely aggressor, while, according to Gilpin (1983, 1988), it is more likely that the hegemon will start a preventive war to keep the status quo. The preventive incentives have been formalized in depth by a recent theoretical literature on exogenous power shifts and commitment problems, which posits war can stop or, at least, slow down a re-allocation of power (see, for example, Powell, 2012, and the references therein) but additional theory is needed to formalize the role of conflict as a catalyst for change in existing power relations, as well as the importance of the mismatch between multiple forms of power, both ex-ante and after wars, for the determination of initiation and continuation incentives.

Countries (in interstate disputes) and ethnic groups (in domestic disputes) decide whether to wage a war or not based not only on the effects on the current distribution of revenues, but also thinking about the effects on future distribution of relative power.[1] After a war, the winner typically obtains control on more weapons, as well as on more resources or institutions that determine indirectly the distribution of resources. In other words, a victory determines both a shift in military power and a shift in political power, where the latter is a catch-all term we use to encompass all the means to obtain a higher share of resources (for example, through a greater control of the democratic institutions). Beside the fact that conflict has effects on both types of power, the two types of power also play different roles *before* wars. While only military power matters for the odds of winning a given war, the relative political power before a war acts as a status quo or outside option to war. The value of winning a war is determined by the degree to which victory affects political power (which determines

[1]For early formal hints about the role of future considerations, see Garfinkel and Skaperdas (2000).

the future outside option), as well as military power (which affects the likelihood of winning future wars).

The model that we introduce is the simplest possible model to address all such issues. We consider a two period model that can be described informally as follows: given any initial distribution of military and political power between two players, and given a random draw of a cost of conflict, the players choose (sequentially, in order to avoid wars simply due to coordination problems) whether to go to war or not; if peace prevails in the period, then the players consume their relative political power; while, if war takes place, the winner consumes the whole surplus of this period minus the conflict costs, and both types of powers have a shift going into the next period. In the second period, starting with the relative power endowments determined in the previous period, there is a new random draw of conflict costs, and then again players choose whether to accept the current political power (and, hence, a peaceful distribution of surplus) or else go to war in order to consume all the surplus of that period, minus the new cost of war.

We first characterize how relative powers affect war onset and duration in this benchmark model with exogenously fixed power shifts in case of war. Then, we extend the analysis to include two recognized ways in which political power can be adjusted endogenously, namely, allowing for bargaining within a period and allowing the winner of a war to choose her desired political power shift. Our first result, in the static as well as in the dynamic case, is that the key cause of war is what we call the *mismatch* between military and political power.[2] In other words, what matters for the probability of a war onset is not whether any type of power is balanced, but rather whether the two types of powers are similarly distributed. Winning

[2]The basic idea that the mismatch between relative powers matters for conflict onset is present also in Filson and Werner (2002). At the same time, the model they study differs from ours in significant ways, in particular for what regards dynamic incentives: in their game, only one player can attack; players care about the division of a single flow of resources and the game ends as soon as these resources is assigned; political power (which they refer to as "benefits") and military power (which they refer to as "military ability") are constant and unaffected by conflict; war affects one player's belief about the (constant) military power, as well as both players' capacity to sustain another battle (what they refer to as "military resources", which also enter players' utility functions in a way which is akin to a discount factor).

a war matters both in terms of acquired resources and in terms of enhancing security going forward.[3] Two super military powers constitute a likely peaceful duo if their political power is also balanced; and two countries in a preponderant imbalance of military power may be equally peaceful if the political power imbalance is proportional to the military one. On the other hand, when there is a mismatch between powers there must be one of the two players that (for sufficiently low realization of the cost of war) is tempted by a war.[4]

Beside confirming the intuition of Filson and Werner (2002) on the role of the mismatch in our model with multiple potential attackers and time-varying power, our model yields predictions also on the "duration" of wars.[5] We capture "duration" of wars in the model by looking at the equilibrium probability of a second war conditional on a war taking place in the first period. We show that duration depends in a nuanced way on the balance of military power, the balance of political power, and on the parameters capturing the size of power shifts after conflict. We show, in particular, that the war is more likely to continue (i.e., the static incentive for war increases) if the politically under-represented player is the militarily advantaged player and if the political power is more unbalanced than the military power.

Almost all the existing literature on duration is on civil wars. They emphasize as potential causes of duration either technology (see, e.g., Balcells and Kalyvas, 2014), inequality (Collier, Hoeffler and Soderbom, 2004), leaders' characteristics (see, e.g., Prorok, 2018), role

[3]As Liberman (1993) puts it, spoils of conquest can be significant and cumulative sequences of wars should be expected.

[4]The fact that the likelihood of war onset depends on the mismatch between political and military power has also implications for the debate on the role of inequality for civil war onset: grievances about inequality are basically captured by a preponderance of political power for one group over another, but our prediction is that such an inequality can lead to war only if it is not paralleled by a similar imbalance in military power. See for example Cederman, Weidmann and Gleditsch (2011) for the empirical findings on the importance of between groups inequality for civil conflict. Huber and Mayoral (2013) find instead that between group inequality is not an important driver of war, and, at the light of our theory, it would be interesting to check whether, in their data, the role of between group inequality becomes relevant again when interacted with some measure of mismatch.

[5]Filson and Werner (2002) present some results on duration but their assumptions limit the scope for a truly dynamic analysis and the underlying mechanism is different. First, they assume the sole potential attacker has resources for at most one war and a second war is, thus, impossible by assumption if this player loses. Second, the game starting after a first battle has occurred is identical to the initial one, up to the potential attacker's beliefs on military power; if this player wins the first war, her confidence in her constant strength grows and, since political power is also unaffected, this always leads to a second war.

of allies and presence of oil (see, e.g., Wiegand and Keels, 2019), stability and credibility of the executive (Thyne, 2012), and ethnopolitical exclusion (Wuckertfennig et al, 2012). The main finding on the duration of interstate wars is in Bennett and Stam (1996), where symmetry of military power emerges empirically as a key factor. At the theoretical level, a paper that explains why parity of observable capabilities might induce longer wars is Slantchev (2004).

Our model points out a non-monotonicity in the role of balance of military power: the cases where both powers are balanced and the cases in which one group is proportionally more powerful in both spheres should be equally unlikely to display wars ex-ante, but the case with balanced military powers could have larger duration in case a first war erupts. The case of balanced military power *together with* unequal political power can be explosive (under some conditions) both in terms of ex ante probability and in terms of duration.

The Houthis in Yemen are a good example of a politically under-represented group with significant military strength.[6] The civil war they started in 2014 can be attributed to both an enduring political and economic discrimination (see Shuja al-Deen, 2019) and their increased strength due to pro-Saleh wings of the country's armed services turning on the Houthi side. President Hadi's government is not only facing a centrist conflict with the Houthis, but he also needs to face a secessionist movement with a now much stronger Southern Transition Council (STC) helped also by the U.A.E., making again a clear case of powers' mismatch (see Salisbury, 2018). Having to face a centrist challenge by the Houthis and a secessionist challenge by the STC, the government is made weaker on each front and unable to make concessions to one without upsetting the other, hence avoiding the mismatch on both sides would be close to impossible even if Hadi wanted. This combination of bargaining difficulties that perpetuate and exacerbate the mismatches makes the Yemen case an unfortunate showcase for why mismatches can persist and wars can be hard to stop.

[6]Salisbury (2017) gives a detailed account of the much higher relative strength of Houthis with respect to their relative political power. See also, for example, Fattah, 2010, and Salmoni et al, 2010, for the various insurgency wars before the bargaining attempt made by President Hadi.

Standard theories of war onset take the relative power of players as a fixed parameter.[7] There are only two strands of literature where power is in some sense endogenous: one is the literature on strategic militarization, where the endogenous power we are talking about is the relative militarization levels chosen by the players in anticipation of war (Powell 1993, Jackson and Morelli 2009, Meirowitz and Sartori 2009). The other strand, closer to us, focuses more explicitly on power shifts. In Powell (2013) and Debs and Monteiro (2014) wars can occur to avoid an anticipated power shift that could be caused by a militarization strategy. In Powell (2013) wars can be followed by a continuation of the game, but only if the outcome of a war is a stalemate. Moreover power shifts are basically a "substitute" of war, rather than being its consequence, as we posit. A related contribution is Yared (2010), which displays repeated wars in equilibrium, but wars are always triggered by an aggressive country in the absence of public information, hence the setting is completely different.

The paper is organized as follows: in Section 2 we introduce our basic model. Section 3 contains all the main equilibrium predictions. Section 4 extends the basic model in the direction of allowing endogenous changes in political power, either through bargaining or making the new relative political power after a conflict be decided by the winner. Section 5 contains some interesting correlations for interstate wars. Section 6 offers some general conclusions. All proofs are relegated to the appendix.

2 Model

Consider the standard problem of two players (ethnic groups or countries, modeled as unitary actors), A and B, having to share a per period surplus (normalized to 1), which may come from exploitation of the local resources or control of the state. The two players' *relative power* is multidimensional: at the beginning of period $t = \{1, 2\}$, player A has a share of *military power* equal to $m_t \in [0, 1]$ and a share of *political power* equal to $p_t \in [0, 1]$; player

[7]See Fearon (1995) and Jackson and Morelli (2011) for a recent survey of rationalist theories of war.

B's military and political power in the same period are, respectively, $(1 - m_t)$ and $(1 - p_t)$.

In each period, the two players sequentially choose whether to wage war against the opponent or not, in random order. If neither player attacks, there is peace and the outcome is determined by relative political power in the current period, that is, by the allocation of resources dictated by peace-time institutions: A consumes p_t and B consumes $(1 - p_t)$. If, instead, either player attacks, a war occurs and the outcome is determined by relative military power in the current period: A wins with probability m_t and B wins with probability $(1 - m_t)$; if A (B) wins, A (B) consumes $1 - c_t$ and B (A) consumes $-c_t$, where $c_t \in [0, 1]$ is the cost of war in period t. This cost is a random variable, drawn from a uniform distribution with support $[0, 1]$ and observed by both players at the beginning of period t. This means that players know the realization of c_t but only know the distribution of c_{t+1} when deciding whether to attack or not in period t.

If the first period is peaceful, the distribution of military and political power at the beginning of the second period is unchanged. On the other hand, war alters the distribution of future military and political power in favor of the winning side. In particular, *power shifts* are a function $f : [0, 1]^2 \to [0, 1]^2$ which maps (m_t, p_t), that is, the initial values of military and political power, to (m_{t+1}, p_{t+1}), that is, the new values of military and political power. We make the following assumption on f.

Assumption 1 *Power shifts take the following form:*

$$m_{t+1} = \begin{cases} m_t + a_t & \textit{if } A \textit{ wins} \\ m_t - a_t & \textit{if } A \textit{ loses} \end{cases} \quad , \quad p_{t+1} = \begin{cases} p_t + b_t & \textit{if } A \textit{ wins} \\ p_t - b_t & \textit{if } A \textit{ loses} \end{cases}$$

where $a_t = g\left(1/2 - |m_t - 1/2|\right)$ *and* $b_t = g\left(1/2 - |p_t - 1/2|\right)$, *for any* $g \in (0, 1)$.

The functional form we adopt for power shifts has many desirable properties, which we list here:

1. **Boundedness:** political and military powers always remain within the boundaries of the interval $[0, 1]$.

2. **Anonymity:** the extent to which power shifts favor the winning side does not depend on the winner's identity but is only a function of starting military and political power (im)balance.

3. **Symmetry:** positive and negative power shifts are symmetric, that is, they have the same magnitude.

4. **Independence:** the power shift in one dimension does not depend on the power shift in the other dimension.

5. **Power Ranking Preservation:** assumption 1 guarantees a *power ranking preservation* property, as proven by the following lemma.

Lemma 1 *In case of conflict, the ranking of A's relative powers is preserved regardless of the conflict's outcome, that is,* $m_t > p_t \Longrightarrow m_{t+1} > p_{t+1}$.

Finally, some properties regarding the magnitude of the power shifts:

Role of g: As g increases, power shifts increase in magnitude: when g approaches 0, there is no change in either power after conflict; when g approaches 1, the shift is maximal and equal to $\min\{(1-m_t), m_t\}$ for military power and to $\min\{(1-p_t), p_t\}$ for political power.

Hegemony vs balance: For any given g, the power shift in any dimension is larger for intermediate initial values of A's relative power in that dimension (that is, military or political *power balance*), and fades to zero for extreme initial values of A's relative power in that dimension (that is, military or political *power imbalance*, namely *hegemony*). This captures the idea that the scope for a change in relative power depends on the initial conditions and can be different in different dimensions.

For ease of exposition, in the remainder of the paper we make the following assumption.

Assumption 2 *At the beginning of the game, A is the politically under-represented or marginalized player that is, $(m_1 - p_1) \geq 0$.*

Note that this assumption is without loss of generality: the analysis for the case where $(m_1 - p_1) < 0$ is identical up to the players' labels. Finally, the following definitions will prove helpful in discussing the intuition behind the results presented in the following Section.

Definition 1 *Let $M_t = (m_t - p_t) \in [0, 1]$ be the degree of A's political under-representation or, in other words, the mismatch between relative military and political powers in period t.*

Definition 2 *Let $\Omega_t = (a_t - b_t)$ be the difference between the change in m_t and the change in p_t after war occurs in period t. In other words, Ω_t is the shock to the powers' mismatch after war occurs in period t.*

Endowed with this definition, we can express the mismatch in period $t + 1$ compactly, as follows:

$$M_{t+1} = \begin{cases} M_t + \Omega_t & \text{if } A \text{ wins} \\ M_t - \Omega_t & \text{if } A \text{ loses} \end{cases}$$

Note that Ω_t can be either positive or negative, depending on the initial degree of power (im)balance in the two dimensions. Regardless of the sign of Ω_t, the power ranking preservation property of power shifts (Lemma 1) implies that, if $M_t \geq 0$, then $M_{t+1} \geq 0$. This means that, if A is the politically under-represented player at the beginning of the game, it will be the politically under-represented player in both periods, regardless of whether conflict occurs in the first period and, when it does, of the winner's identity.

3 Results

3.1 Probability of War in Static Game

Consider first the static game (or the last period in the dynamic game). When players only care about their current consumption, they decide whether to attack or not by comparing the share of resources assigned to them by peace-time institutions — which depends on relative political power in the current period — and the expected consumption after conflict — which is a function of relative military power and the cost of war in the current period. In particular, A prefers a peaceful outcome if and only if:

$$p_t > m_t - c_t \quad \Longleftrightarrow \quad c_t > m_t - p_t = M_t$$

Similarly, B prefers peace if and only if:

$$(1 - p_t) > (1 - m_t) - c_t \quad \Longleftrightarrow \quad c_t > p_t - m_t = -M_t$$

which is always the case, since $c_t \geq 0$ and $M_t \geq 0$.

Combining the two conditions above, we have war when:

$$c_t \leq m_t - p_t = M_t$$

The probability of war (before the realization of c_t) is, thus:

$$\Pr(c_t \leq M_t) = M_t$$

Proposition 1 *In the absence of concerns for the future, only the politically under-represented player has an incentive to attack and the ex-ante probability war occurs is equal to the mismatch between political and military power, that is, M_t.*

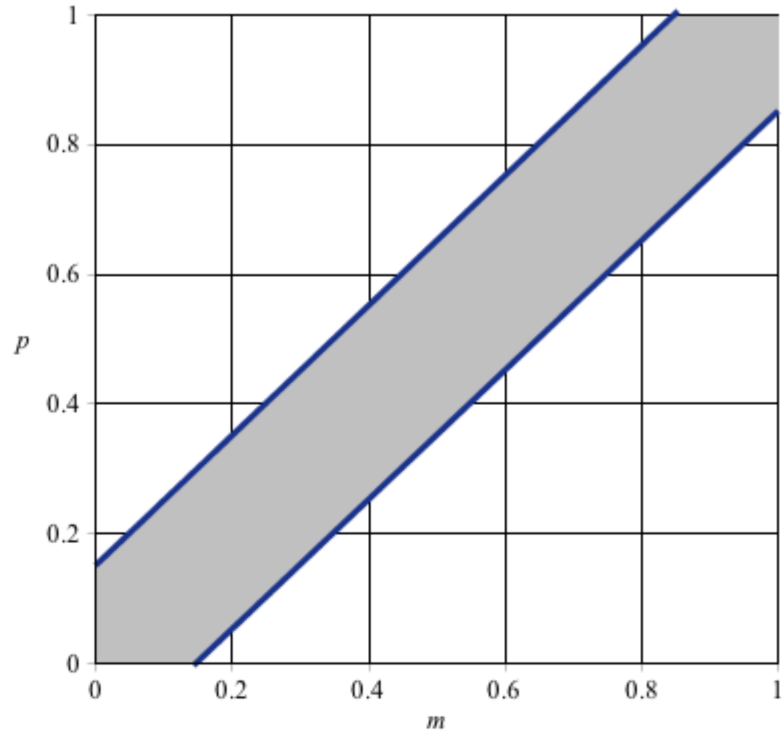

Figure 1: War and peace regions in the static game. The shadeqd area is the set of all power states where there is peace, given a realized cost of war of 0.15.

When Assumption 1 is violated (that is, when $m_t < p_t$ and, thus, B is the politically under-represented player), a similar analysis shows that A never attacks and that the chance B attacks is equal to $(p_t - m_t) = -M_t$, that is, the mismatch between political and military power. Figure 1 shows the power states for which war is predicted to occur in the static game for a given value of c_1.

3.2 Value Functions (Expected Utilities from Power Allocations)

The equilibrium strategies and the associated probability of war for the static case apply to the second period of the two-period game. Consequently, we can use them to compute the *value functions* of state (m_2, p_2) — that is, the expected utilities players A and B derive from power allocations (m_2, p_2), evaluated at the end of the first period.

$$
\begin{aligned}
V^A(m_2, p_2) &= (1 - M_2)\, p_2 + M_2 \int_0^{M_2} \frac{m_2 - c}{M_2} dc = (1 - M_2)\, p_2 + M_2 \left(m_2 - \frac{M_2}{2} \right) \\
V^B(m_2, p_2) &= (1 - M_2)\, (1 - p_2) + M_2 \left(1 - m_2 - \frac{M_2}{2} \right)
\end{aligned}
$$

In both expressions, the first term is the value of the state in peace — that is, a player's relative political power — weighted by the probability of peace in that state, $1 - M_2$. The second term is the value of the state in war, weighted by the probability of war in that state, M_2. The value of the state in war consists of the expected consumption after war — that is, a player's relative military power — minus the expected cost of war, $M_2/2$.

We can further simplify the two players' value functions and write them as follows:

$$
\begin{aligned}
V^A(m_2, p_2) &= p_2 + \frac{(M_2)^2}{2} \\
V^B(m_2, p_2) &= (1 - p_2) - 3\frac{(M_2)^2}{2}
\end{aligned}
$$

The value functions are essential elements of the dynamic incentive to wage war in the first period of the two-period game so they are worth a close look. We highlight some of their properties. First, the value function of A, the politically under-represented player, is increasing in the powers' mismatch, while the value function of B, the politically over-represented player is decreasing in the powers' mismatch. The intuition is the following. Regardless of its relative military power (and, thus, its chance to win a war), the politically under-represented player expects a gain in consumption from conflict (with respect to its consumption in peace), even when taking into account the cost of war. Both this expected gain — which is equal to the mismatch minus the expected cost of war, that is, $M_2 - M_2/2$ — and the chance of realizing it — which is equal to the probability of war, that is, M_2 — grow with the magnitude of the mismatch. On the other hand, the politically over-represented player expects a loss in consumption from conflict (with respect to its consumption in peace).

Both this expected loss and the chance of realizing it are increasing in the mismatch. Second, the value functions of both players are always increasing in a player's relative military power: for any level of A's relative political power, A prefers a larger m_2 and, thus, a larger mismatch, while B prefers a lower m_2 and, thus, a smaller mismatch. Third, the value function of A, the politically under-represented player, is increasing in its own relative political power. On the other hand, the value function of B, the politically over-represented player, is not monotonic in its own relative political power: for a given m_2, V^B is increasing in $(1 - p_2)$ if and only if $(1 - p_2) < (1 - m_2) + \frac{1}{3}$. Hence, if $(1 - m_2) < 2/3$, B's value function is decreasing in $(1 - p_2)$ for high enough $(1 - p_2)$. Intuitively, if B's relative military power is sufficiently small, B prefers not to have too much political power as A might engage in a war that B is likely enough to lose. Finally, total expected welfare in the second period is equal to:

$$W(m_2, p_2) = V^A(m_2, p_2) + V^B(m_2, p_2) = 1 - M_2$$

which intuitively goes to 1, the total per period surplus, only when the chance of war is zero, that is, when there is no mismatch. It then decreases as the chance of war increases.

3.3 Probability of War in Dynamic Game

Having analyzed players' behavior in the second period of the game, we now move to players' behavior in the first period. We assume that players discount future consumption with a common factor $\delta \in [0, 1]$. When $\delta = 0$, players are myopic and the analysis for the first period coincide with the one for the static game above. More interestingly, when $\delta > 0$, players are forward-looking and take into account how their current decision affects the future distribution of relative powers and, thus, their future outcomes.

In the first period, A prefers to wage a war against B rather than maintain peace if:

$$EU_A(Peace) \quad < \quad EU_A(War)$$

$$p_1 + \delta V^A(m_1, p_1) \quad < \quad (m_1 - c_1) + \delta \left(\begin{array}{c} m_1 V^A(m_1 + a_1, p_1 + b_1) + \\ (1 - m_1) V^A(m_1 - a_1, p_1 - b_1) \end{array} \right)$$

In case of peace, A consumes p_1 today and its future military and political power coincide with the current ones; thus, its expected utility from the second period is given by the value function evaluated at (m_1, p_1). In case of war, A's expected consumption today is $m_1 - c_1$ and its future military and political power will shift depending on the war outcome; if A wins (which happens with probability m_1), A's military and political power grow and its expected utility from the second period is given by the value function evaluated at $(m_1 + a_1, p_1 + b_1)$; if, instead, A loses (which happens with probability $1 - m_1$), A's military and political power decrease and its expected second-period utility is given by the value function evaluated at $(m_1 - a_1, p_1 - b_1)$. Substituting the value functions, we have:

$$p_1 + \delta \left(p_1 + \frac{(M_1)^2}{2} \right) \quad < \quad (m_1 - c_1) + \delta \left(\begin{array}{c} m_1 \left(p_1 + b_1 + \frac{(M_1 + \Omega_1)^2}{2} \right) + \\ (1 - m_1) \left(p_1 - b_1 + \frac{(M_1 - \Omega_1)^2}{2} \right) \end{array} \right)$$

$$c_1 \quad < \quad \overline{c}_A := M_1 + \delta(2m_1 - 1)b_1 + \delta \left(\frac{(\Omega_1)^2}{2} + (2m_1 - 1) M_1 \Omega_1 \right)$$

Namely, A has an incentive to wage a war if the realized cost of war in the first period, c_1, is below threshold $\overline{c}_A$. The first term in $\overline{c}_A$ is the *static* incentive to attack, that is, the powers' mismatch M_1 in the first period. The second and the third term represent the *dynamic incentive to attack,* weighted by the importance of the future, δ. In particular, the second term represents the expected impact of first-period war on the second-period peace dividend, that is, $E[p_2|War] - E[p_2|Peace] = E[p_2|War] - p_1$. This is positive — and, thus, it increases A's incentive to attack with respect to the static case — if and only if A is militarily advantaged (that is, when $m_1 > 1/2$) and, thus, political power is more likely than not to shift to its advantage. The third term represents the expected impact of the first-period war on the marginal gain from a second-period war relative to peace (considering the expected cost of war), weighted by the expected chance of this happening, that is:

$$\mathrm{E}\left[M_2|War\right]\mathrm{E}\left[M_2-\frac{M_2}{2}|War\right]-\mathrm{E}\left[M_2|Peace\right]\mathrm{E}\left[M_2-\frac{M_2}{2}|Peace\right]=\frac{\mathrm{E}\left[\left(M_2\right)^2|War\right]-\left(M_1\right)^2}{2}.$$

This is strictly positive when $m_1 \geq 1/2$ and, by continuity, is strictly positive also when A is militarily disadvantaged and likely to lose the war, as long as m_1 is sufficiently close to 1/2. In the proof of Proposition 2, we show that this term is positive as long as $m_1 \geq \frac{2-g}{4}$ and negative otherwise.

Similarly, B prefers to wage a war against A rather than maintain peace if:

$$\begin{aligned}
EU_B(Peace) &< EU_B(War) \\
1-p_1+\delta V^B(m_1,p_1) &< 1-m_1-c_1+\delta\left(\begin{array}{c} m_1 V^B(m_1+a_1,p_1+b_1)+ \\ (1-m_1)V^B(m_1-a_1,p_1-b_1)\end{array}\right) \\
c_1 &< \bar{c}_B := -M_1-\delta(2m_1-1)b_1-3\delta\left(\frac{\left(\Omega_1\right)^2}{2}+(2m_1-1)M_1\Omega_1\right)
\end{aligned}$$

Hence, B has an incentive to go to war if c_1 is below the following threshold $\bar{c}_B$. The cost thresholds below which A and B, respectively, prefer to wage war simplify to:

$$\bar{c}_A = M_1+\delta\left[\left(2m_1-1\right)\left(b_1+M_1\Omega_1\right)+\frac{\left(\Omega_1\right)^2}{2}\right] \tag{1}$$

$$\bar{c}_B = -M_1+\delta\left[-\left(2m_1-1\right)\left(b_1+3M_1\Omega_1\right)-3\frac{\left(\Omega_1\right)^2}{2}\right] \tag{2}$$

Since the cost of war is non-negative, player A has a potential incentive to attack (that is, there exists a realization of c_1 such that this player prefers war to peace) only if $\bar{c}_A$ is positive, and likewise for player B. For positive values of its cost threshold, the ex-ante probability that a player attacks (that is, the probability that the realization of c_1 is below this threshold) is increasing in the cost threshold.

How do dynamic considerations change players' incentives to attack the opponent? Consider player A, the under-represented or marginalized player. The term in square brackets of the RHS of equation (1) represents A's dynamic incentive to attack. When this term is positive, a forward-looking player A has a greater incentive (and, thus, a greater ex-ante probability) to attack than a myopic player A. This happens whenever $m_1 \geq 1/2$, that is, whenever A has a military advantage. However, A's dynamic incentive can be positive even when it expects to lose a war (that is, when $m_1 < 1/2$), as long as m_1 is sufficiently close to 1/2. Finally, when m_1 is sufficiently close to 0, A's dynamic incentive is negative. When this is the case, a forward-looking player A has a smaller incentive (and, thus, a smaller ex-ante probability) to attack than a myopic player A. When the dynamic incentive is negative and its magnitude (weighted by the discount factor) is larger than the static incentive, a forward-looking player A never wages war while there is a positive chance that a myopic player A does so. This happens as long a m_1 is sufficiently small and δ is sufficiently high. Now consider player B, the over-represented player. The second term of the RHS of equation (2) represents B's dynamic incentive to attack. This is negative when player B is militarily disadvantaged (i.e., $m_1 > 1/2$) but is positive when m_1 is sufficiently small. In particular, when m_1 is sufficiently close to 0, then B's dynamic incentive to attack more than compensates the static disincentive and war becomes an attractive option when the cost of conflict is low.

Propositions 2 and 3 summarize the above discussion.

Proposition 2 *The politically under-represented player (A)'s dynamic incentive to attack is positive (and, thus, the probability it attacks grows with δ) if $m_1 > m^* \in \left(\frac{2-g}{4}, \frac{1}{2}\right)$ and negative otherwise. When A is sufficiently forward looking and sufficiently military disadvantaged, the dynamic incentive dominates the static one and, contrary to the static case, A does not have any incentive to attack.*

Proposition 3 *The politically over-represented player (B)'s dynamic incentive to attack is positive (and, thus, the probability it attacks grows with δ) if $m_1 < m^{**} \in \left(\frac{2-g}{4}, \frac{1}{2}\right)$ and negative otherwise. When B is sufficiently forward looking and sufficiently military advantaged, the dynamic incentive dominates the static one and, contrary to the static case, B has an incentive to attack.*

Proposition 4 presents results on the role of the mismatch in the dynamic game. In its proof, we show that, in this case, the existence of an initial mismatch between powers is not strictly necessary for war onset, as long as military power is not perfectly balanced. This is due to the expected increase in the future peace-time dividend anticipated by the militarily advantaged player. Nonetheless, even in the dynamic case, the initial misalignment of political and military power is an important source of conflict and its effect is actually magnified by dynamic considerations for most initial allocations of power.

Proposition 4 *When players are forward looking (that is, $\delta > 0$), the ex-ante probability of war is strictly positive even when $M_1 = 0$ as long as $m_1 \neq 1/2$. Nonetheless, the ex-ante probability the politically under-represented player (A) attacks is strictly increasing in the magnitude of the mismatch. Moreover, A's dynamic incentive to attack is increasing in the mismatch for the vast majority of initial power allocations — the exception being cases where $m_1 \in (1/2, 5/6)$ and $p_1 \in (0, 1/6)$ — amplifying the static incentive.*

3.4 War Duration

In addition to investigating the likelihood of war onset as a function of the initial allocation of military and political power, our framework allows us to make progress on an important yet understudied question in international relations, that is, why some conflicts last longer than others. Before stating our results, we need to introduce one additional definition.

Definition 3 *Let $\mu_t = \left|m_t - \frac{1}{2}\right|$ be the degree of imbalance in military power in period t. Similarly, let $\pi_t = \left|p_t - \frac{1}{2}\right|$ be the degree of imbalance in political power in period t. When*

$\mu = 0$ ($\pi = 0$) there is perfect balance of military (political) power, whereas if $\mu = 1/2$ ($\pi = 1/2$) one player has hegemony in military (political) power.

The next proposition characterizes the chance of future war conditional on the initial degree of power imbalances and the identity of the winner in the current war.

Proposition 5 *If $\pi_1 > \mu_1$, then the incentive to wage a second war (i.e., the static incentive to fight) grows when A, the politically under-represented player, wins the first conflict. On the other hand, if $\pi_1 < \mu_1$, then the incentive to wage a second war grows when B, the politically overrepresented player, wins the first conflict.*

The following proposition characterizes war duration as a function of initial conditions:

Proposition 6 *Conditional on starting, a war is more likely to last for longer (i.e., to continue for a second period) if political power is initially more unbalanced than military power and the politically under-represented player is militarily advantaged.*

The intuition behind these results is that, when the initial allocation of military power is more balanced than the initial allocation of political power, then military power will change more than political power after a war. As a consequence, the powers' mismatch will grow if the politically under-represented wins a war and will decrease if it loses it. When the politically under-represented player is militarily advantaged, and thus expected to win, war will likely continue to a second period. This is due to the fact that the mismatch between political and military power is expected to persist (in fact, it is expected to grow) and a successful conflict will not reduce the grievance of the politically under-represented player (in fact, it will exacerbate it).

4 Extensions

In this Section, we consider two extensions, both in the direction of allowing political power to be endogenously adjusted. In Section 4.1, we allow for the endogenous allocation of current resources in the absence of conflict (that is, p_1) via the standard possibility of bargaining, whereas in Section 4.2 we allow the winner of a conflict to choose the new surplus sharing institutions (that is, p_2) rather than assuming an exogenous power shift. The goal is to show that the importance of the initial mismatch between military and political power is robust to these extensions.

4.1 Bargaining

For the extension to bargaining, we focus on the static game and consider the case where only one player has incentives to attack, that is, the potential grievances about the status quo can come only from one side — a group out of power or a country having the lower end in any international dispute.

In order to introduce bargaining in a classic way, we extend the model to include asymmetric information on relative strength. Specifically, we assume that the player favored by the status quo is uncertain about the true strength of the opponent in case of conflict.[8]

Consistent with the previous sections, let player B be in power and player A be the potential challenger. We denote by p the status quo share of the surplus to player A, and c is the cost of conflict. The assumption that B is uncertain about A's strength can be represented by the simple case with two types: in case of conflict, the probability A wins is m_h if A is *strong* and $m_l < m_h$ if A is *weak*; A's strength is her private information but B knows the distribution of types in the population: A is strong with probability $q \in (0, 1)$ and weak with probability $(1 - q)$. We assume that $m_h > m_l > p$. The timing of the game is as follows:

1. Nature draws c from a uniform distribution with support $[0, \overline{c}]$;

2. A observes c and her strength, decides whether to challenge the status quo p or not;

[8]Asymmetric information is one of the well known sources of bargaining breakdown leading to conflict (see Fearon 1995 and Jackson and Morelli 2011).

3. if faced with a challenge, B decides which revision of the status quo to propose, r;

4. A accepts or rejects, and in the latter case conflict starts.

For the analysis, we need to consider three cases, depending on the realization of c.

Case 1: Consider first the relevant case in which the realization of c is below $m_l - p$, which, ex ante, happens with probability $\frac{m_l - p}{\bar{c}}$. In this case, both the strong and the weak A types have incentive to challenge the status quo (i.e., we have a *pooling* equilibrium). Hence, B's equilibrium belief that the challenger is of type h is exactly q. B can make a take-it-or-leave-it offer to A without knowing her strength, where the offer takes the form of a revised share r to replace p. It is well known, as a simple application of the Myerson-Satterthwaite theorem (Myerson and Satterthwaite 1983), that, for any realization of c, there exists q^* such that for any $q < q^*$ the offer by B will be a risky offer that would appease A in case her relative strength is m_l but not if her relative strength is m_h. The expression for the threshold as a function of c is

$$q^*(c) := \frac{m_h - m_l}{m_h - m_l + 2c}$$

and conflict would then be an equilibrium phenomenon (conditional on $c < m_l - p$) with probability q whenever $q < q^*(c)$.

Case 2: When $c \geq m_h - p$, A does not challenge the status quo regardless of her type.

Case 3: The most complex case is the one where $c \in [m_l - p, m_h - p)$. If $q > q^*(c)$, we have a pooling equilibrium where both A types challenge the status quo and B offers revision, $r_h = m_h - c$, which is accepted by both types and leads to a peaceful period with certainty. When $q < q^*(c)$, this cannot be an equilibrium: if A types pool and both challenge the status quo, B offers $r_l = m_l - c$; but this makes the low A type deviate to no challenge and, thus, cannot be part of an equilibrium. The low type never challenging the status quo (the other potential pure strategy equilibrium) cannot be an equilibrium either: in this case, B's equilibrium belief he is facing a strong A type when the status quo is challenged is 1, hence above $q^*(c)$, and B's offer would then be r_h; but this, in turn, creates an incentive to deviate by the low type who would want to challenge. Thus, when $c \in [m_l - p, m_h - p)$ and $q < q^*(c)$, the equilibrium must be in mixed strategies. More precisely, it is a *semi-pooling* equilibrium where a strong A type challenges with probability 1 whereas a low type challenges with probability $\sigma \in (0,1)$, making B indifferent between offering r_h and offering r_l. Thus, σ must solve

$$q + (1-q)\sigma = q^*(c).$$

Consistently, B should choose r_h with probability τ solving

$$\tau(m_h - c) + (1-\tau)(m_l - c) = p.$$

Since $\frac{\partial \tau}{\partial p} > 0$, the probability the safe offer is proposed (that is, τ) is increasing in p. It follows that, even in this range of parameters, when p is lower (that is, when the powers' mismatch is higher), there is a higher probability of war.

From the point of view of the ex-ante probability of war — that is, evaluating the chance a situation will escalate to conflict before the realization of c — we can conclude that conflict is an equilibrium outcome with a probability that is increasing in the mismatch between A's relative strength and A's relative political power: fixing m_h, m_l, the powers' mismatch increases as p decreases and the above considerations imply that the ex-ante probability of war increases as the mismatch increases. This is because decreasing p increases the probability of war conditional on $q < q^*(c)$ both in Case 1 (where decreasing p leads to a larger range of values of c below a higher $m_l - p$ threshold) and in Case 3 (where decreasing p induces a lower τ), while the range of values of c corresponding to Case 2 without conflict shrinks. This discussion is summarized in the following proposition:

Proposition 7 *For every realization of c such that $q > q^*(c)$ there is always peace in the presence of bargaining. Whenever $q < q^*(c)$ conflict happens in equilibrium with probability q if $m_l - p > c$ and with probability $\frac{q(1-\tau)}{q+\sigma(1-q)}$ if $c \in [m_l - p, m_h - p)$. The probability of war is increasing in the mismatch.*

4.2 Endogenous Power Shifts

The power shifts after a war occurs can be a choice variable for the winner, rather than being determined in an exogenous manner as in our benchmark analysis. In this Section, we analyze wars where the winner of a conflict can choose the shift in political power endogenously. For mathematical convenience and ease of exposition, we assume that military power is fixed and independent of war outcomes and we denote it with m, without time subscripts. In all other respects, the game is identical to the one described in Section 2.

The modified timing of the game is as follows:

1. Players observe c_1 and decide whether to attack or not;

2. if there is a war, the winner chooses $p_2 \in [0, 1]$ given m;

3. players observe c_2 and decide whether to attack or not.

The analysis of the second period is identical to the analysis of the static game in our benchmark case (Section 3.1). When there is a first-period war and player $j = \{A, B\}$ wins, j chooses $p_2^j \in [0, 1]$ to maximize its expected utility from the second period, given the exogenous relative military power. Following the same logic from Section 3.2, we can derive A's and B's objective functions when they get to choose p_2:

$$V^A(m, p_2) = \begin{cases} p_2 + \frac{(m-p_2)^2}{2} & \text{if } p_2 \leq m \\ p_2 - 3\frac{(p_2-m)^2}{2} & \text{if } p_2 > m \end{cases}$$

$$V^B(m, p_2) = \begin{cases} (1-p_2) - 3\frac{(m-p_2)^2}{2} & \text{if } p_2 \le m \\ (1-p_2) + \frac{(p_2-m)^2}{2} & \text{if } p_2 > m \end{cases}$$

Proposition 8 *When the winner of a first-period war chooses endogenously the second-period political power, A's and B's optimal choices are:*

$$p_2^A = \begin{cases} m + 1/3 & \textit{if } m < 2/3 \\ 1 & \textit{if } m \ge 2/3 \end{cases}$$

$$p_2^B = \begin{cases} 0 & \textit{if } m \le 1/3 \\ m - 1/3 & \textit{if } m > 1/3 \end{cases}$$

A player's optimal level of relative political power is weakly greater than its level of relative military power. More interestingly, when political power can be adjusted endogenously, the winner of a conflict does not always exclude the loser completely from the distribution of resources. In particular, this is not the case when the initial military power is balanced. In this case, the winner trades off an increase in its current resources and a decrease in tomorrow's chances of a conflict.

War Duration To study the duration of a war, we investigate the probability of a conflict in the second period, conditional on a conflict in the first period.

Proposition 9 *When political power changes endogenously, the expected mismatch in the second period, conditional on war in the first period is:*

$$E\left[|M_2||War\right] = m\left(p_2^A - m\right) + (1-m)\left(m - p_2^B\right) = \begin{cases} \frac{4m}{3} - m^2 & \textit{if } m < \frac{1}{3} \\ \frac{1}{3} & \textit{if } m \in \left[\frac{1}{3}, \frac{2}{3}\right] \\ \frac{2m}{3} + \frac{1}{3} - m^2 & \textit{if } m > \frac{2}{3} \end{cases}$$

Corollary 1 *When political power changes endogenously, the probability of war in the second period, conditional on war in the first period, is strictly increasing in m for $m \leq 1/3$, constant in m for $m \in [1/3, 2/3]$, and strictly decreasing in m for $m \geq 2/3$.*

This result tells us that the duration of wars in the presence of endogenous power shifts is higher when there is balance of military power. When fighting effort is endogenous, like in the traditional Hirshleifer appropriation technology models, the fact that duration may be higher in more balanced contexts is a natural result, because any war of attrition lasts longer if the two parties in the tug of war have equal strength. However, the novelty of our result is that balance of military power and duration are correlated even when not taking into account efforts, only ex ante incentives.

Probability of War When forward-looking players decide whether to wage a war or not in the first period, they take into account the ability of the winner to determine future relative political power. In the remainder of this Section, we assume that military power is sufficiently balanced, that is, $m \in [1/3, 2/3]$. This allows us to focus on the case where the optimal choice of political power by the winner of a first-period war is interior, rather than a corner solution determined by the relative power feasibility constraint.

The indirect utilities after a victory by A are:

$$\begin{aligned} V^A\left(m, p_2^A\right) &= m + 1/6 \\ V^B\left(m, p_2^A\right) &= 13/18 - m \end{aligned}$$

The indirect utilities after a victory by B are:

$$\begin{aligned} V^A\left(m, p_1^B\right) &= m - 5/18 \\ V^B\left(m, p_1^B\right) &= 7/6 - m \end{aligned}$$

At the beginning of the first period, after c_1 has been realized, player A faces the following expected utilities from peace and from war:

$$
\begin{aligned}
EU^A(Peace) &= p_1 + \delta V^A(m, p_1) \\
EU^A(War) &= (m - c_1) + \delta\left[mV^A\left(m, p_2^A\right) + (1-m)V^A\left(m, p_2^B\right)\right]
\end{aligned}
$$

where p_2^B is the political power optimally chosen by B, p_2^A is the political power optimally chosen by A, the value function in case of peace is $V^A(m, p_1) = (1 - M_1)p_1 + M_1\left(m - \frac{M_1}{2}\right) = p_1 + \frac{(M_1)^2}{2}$ as characterized in Section 3.2, and the value functions in case of war are the ones characterized above. As in previous sections, we assume that A is the marginalized side at the beginning of the game, that is, $m > p_1$. This means that the initial powers' mismatch (that is, the probability of a second-period war if there is no first-period war) is equal to $M_1 = m - p_1$.

A prefers to wage a war against B rather than maintain peace if:

$$
\begin{aligned}
EU^A(Peace) &< EU^A(War) \\
p_1 + \delta V^A(m, p_1) &< (m - c_1) + \delta\left[mV^A\left(m, p_2^A\right) + (1-m)V^A\left(m, p_2^B\right)\right]
\end{aligned}
$$

Plugging in the value functions, we have:

$$
\begin{aligned}
p_1 + \delta\left[(1 - M_1)p_1 + M_1\left(m - \frac{M_1}{2}\right)\right] &< (m - c_1) + \delta\left[m\left(m + \frac{1}{6}\right) + (1-m)\left(m - \frac{5}{18}\right)\right] \\
c_1 &< c_A^* := M_1 + \delta\left[M_1 - \frac{(M_1)^2}{2} + m\frac{4}{9} - \frac{5}{18}\right]
\end{aligned}
$$

A does not have any incentive to attack B (that is, there is no realization of $c_1 < c_A^*$) only when $c_A^* < 0$. This happens only when the dynamic incentive is negative and more than compensates the static incentive. The only term which can take negative values in c_A^* is $\delta\left(m\frac{4}{9} + \frac{5}{18}\right)$ and this attains its minimum at $-\frac{7}{54}$, when $\delta = 1$ and $m = 1/3$. A sufficient condition for c_A^* to be positive for any $\delta \in [0,1]$, any $m \in [1/3, 2/3]$, and any $p_1 \in [0, m]$ is $M_1 > 1/15$ which guarantees the sum of the other three terms is always greater than $\frac{7}{54}$. How does the incentive for A to attack B change with the size of the mismatch? Consider an increase in m, keeping p_1 fixed. This amounts to an increase in the initial powers' mismatch, $M_1 = m - p_1$. As discussed in Section 3.1, A's static incentive to attack B is increasing in the size of the mismatch. When the shift of political power is endogenous, also A's dynamic incentive to attack is increasing in the size of the mismatch. This follows from the fact that $M_1 + \frac{(M_1)^2}{2}$ is strictly increasing in M_1.

Similarly, B prefers to wage a war against A rather than maintain peace if:

$$
\begin{aligned}
EU^B(Peace) &< EU^B(War) \\
(1-p_1) + \delta V^B(m, p_1) &< 1 - m - c_1 + \delta\left[mV^B\left(m, p_2^A\right) + (1-m)V^B\left(m, p_2^B\right)\right] \\
(1-p_1) + \delta\left[(1-p_1) - 3\frac{(M_1)^2}{2}\right] &< 1 - m - c_1 + \delta\left[m\left(\frac{13}{18} - m\right) + (1-m)\left(\frac{7}{6} - m\right)\right] \\
c_1 &< c_B^* := -M_1 + \delta\left[-M_1 + 3\frac{(M_1)^2}{2} + \frac{1}{6} - m\frac{4}{9}\right]
\end{aligned}
$$

B has no incentive to attack A (that is, there is no realization of $c_1 < c_B^*$) when $c_B^* < 0$. The only term in c_B^* which can take positive values is $\delta\left(\frac{1}{6} - m\frac{4}{9}\right)$ which attains its maximum at $\frac{1}{54}$, when $\delta = 1$ and $m = \frac{1}{3}$. A sufficient condition for c_B^* to be negative for any $\delta \in [0,1]$, any $m \in [1/3, 2/3]$, and any $p_1 \in [0, m]$ is $M_1 > 1/100$ which guarantees the sum of the other three terms is always lower than $-\frac{1}{54}$. On the other hand, when $m \in [1/3, 3/8)$, B attacks A with positive probability (that is, $c_B^* > 0$) if δ is sufficiently close to 1 and M_1 is sufficiently close to 0.

This discussion is summarized in the following proposition, which mirrors results from Section 3, see Propositions 2 and 3:

Proposition 10 *When political power changes endogenously, the probability the politically under-represented player attacks is strictly positive when* $M_1 > 1/15$*; on the other hand, there* *is no chance the politically over-represented player attacks when $M_1 > 1/100$. Moreover, the probability the politically under-represented player attacks is weakly increasing in the initial mismatch between political and military power (strictly increasing when $c_A^* \in [0,1]$, which is always the case when $M_1 > 1/15$).*

To summarize the content of this Section, Propositions 7 and 10 show that the probability of war is increasing in the size of the initial mismatch between political and military power even when we allow for the endogenous allocation of current (through bargaining) or future (through conflict) political power.

5 Some Empirical Observations

An empirical test of the mismatch theory that, in different ways, emerges from Filson and Werner (2002) and from our model poses several challenges. The most important ones are, of course, identification and endogeneity. However, we think it is useful to display an interesting set of correlations between powers' mismatch and interstate war onset and incidence, leaving for future work the possibility to establish causality.

Our interstate war dataset includes 273 country dyads for the period 1960–2006. Since most interstate wars are between contiguous countries, our dataset only includes dyads that share either a land boarder or a river boarder.[9] Information on our dependent variables, interstate conflicts' onset and incidence for the years 1816–2001, comes from the Dyadic Militarized Interstate Disputes dataset (Maoz 2005). The main challenge is to find valid proxies of military and political power, that is, using the notation of our model, m and p. As a proxy of m, we construct the ratio of troops using data on countries' military personnel from the COW National Material Capabilities Dataset (NMC, Singer, Bremer, and Stuckey 1972). As a proxy of p, we construct the ratio of GDP using data from the World Bank

[9] Information on territorial contiguity comes from the COW Direct Contiguity Dataset (Stinnett, Tir, Schafer, Diehl, and Gochman 2002) which lists all the country dyads from 1816 to 2006 and their territorial relationship.

National Accounts Data. Our key independent variable, the *mismatch* between military and political power, is then constructed as the absolute value of the difference between these two variables.

Following Cunningham and Lemke (2013)—who show that interstate and intrastate wars are explained by similar factors—we use three sets of control variables, one set drawn from the interstate wars literature and two sets drawn from the civil wars literature. A first set of correlates is from the existing literature on the onset of interstate wars: the number of allies each state in the dyad has; the number of interstate "enduring rivals" each state in the dyad has; the number of direct land neighbors each state in the dyad has; an indicator of whether either country in the dyad is a major power as coded by COW. [10] A second set of correlates is drawn from Hegre and Sambanis (2006) who produce a list of the most robust predictors of civil war onset. Borrowing from their findings, our second set of control variables includes the following: a dummy indicating whether the state experienced a previous interstate conflict; gross domestic product per capita; logged population; and Polity III 0-10 democracy score.[11] Finally, the third set of control variables includes insurgency conditions and ethnicity, as suggested by the canonical study by Fearon and Laitin (2003). The specific correlates are the following: the percentage of the two states' territory that is mountainous, whether the two states have noncontinuous territory, whether they are a new state (a dummy equal to 1 for the first two years after independence), a measure of instability (a dummy equal to 1 when the state's regime type score has changed by three or more in any of the previous three years), and a measure of ethnic fractionalization.[12]

Table 1 presents the results for three separate specifications. Following Cunningham

[10]These variables come from the replication data for Cunningham and Lemke (2013). Their original source is EUGene (Bennett and Stam 2000), with the exception of the number of allies and rivals which come from Diehl and Goertz (2001).

[11]These variables also come from the replication data for Cunningham and Lemke (2013). Their source for gross domestic product per capita and for population is Gleditsch (2002).

[12]Following Fearon and Laitin (2003), Cunningham and Lemke (2013) also include in this third set of controls an indicator of whether a country is an oil exporter. In our current dataset, this information is available only for a subset of observations (6,222) and for this reason in Table 1 we report results without these variables. Including these additional two dummies does not change the results.

and Lemke (2013), we avoid combining all variables in one analysis because of the dubious usefulness of kitchen-sink models.[13] In Table 1 we report odds ratios, interpreted in the following way: odds ratios below 1 mean that increasing values of the independent variable make conflict onset less likely, while values above 1 mean that conflict onset is more likely. Table 1 implies strongly that the mismatch between political and military power is positively and significantly correlated with interstate war onset and with interstate war incidence, even when controlling for all the other relevant factors.

6 Conclusions

This paper proposes a simple theory of war onset and duration that emphasizes the critical role of the mismatch between military and political power. We have then studied the dynamic effects, the robustness to endogenous bargaining breakdown, and preliminary empirical evidence. Rather than summarizing more in detail the various results, we want to conclude by offering some further clarifications about the perspective that we propose for future research. The traditional formal theory approach to understand war is based on bargaining, in the following specific manner: assuming that, in case of conflict, all the surplus goes to the winner, or, in any case, that the winner decides the surplus allocation, the initial distribution of political power, $(p, 1-p)$, is generally considered irrelevant in terms of final payoff division conditional on going to war; it follows that the outside option to a bargaining agreement typically depends only on the distribution of military power, $(m, 1-m)$. For these reasons, the role of the mismatch has never been explicitly emphasized before Filson and Werner (2002), even though it is obviously well understood that a mismatch between outside option expected utilities (that is, conflict expected utilities) and the status quo utilities is a natural ingredient of the likelihood of bargaining break down. We depart from those char-

[13] See also Achen (2002) and Ray (2003), who document how misleading regression models with many regressors can be. A model that includes all the available independent variables gives similar results.

Model	Variables	Conflict Onset	Conflict Incidence
No Controls	Mismatch	4.51***	7.60***
	Intercept	0.05***	0.07***
	Model χ^2	10.12***	15.71***
	Observations	8,509	8,509
Controls from COW	Mismatch	2.74*	5.23***
	Allies A	0.94	0.94
	Allies B	0.94	0.81*
	Rivals A	1.42***	1.39**
	Rivals B	1.76***	2.25***
	Neighbors A	0.92	0.92
	Neighbors B	0.95	0.98
	Major Power	0.54	0.53
	Intercept	0.08***	0.10***
	Model χ^2	166.04***	83.90***
	Observations	8,509	8,509
Controls from Hegre & Sambanis (2006)	Mismatch	5.76***	8.13***
	Prior War A	1.14	1.65**
	Prior War B	1.74**	2.78***
	GDP Per Capita A	0.93*	0.94
	GDP Per Capita B	0.96	0.98
	Log of Population A	1.17**	1.14
	Log of Population B	1.04	1.02
	Democracy A	1.00	0.98
	Democracy B	1.07	1.09*
	Intercept	0.01***	0.02***
	Model χ^2	41.32***	52.69***
	Observations	7,650	7,650
Controls from Fearon & Laitin (2003)	Mismatch	4.74***	7.35***
	Log of Mountains A	1.19*	1.18
	Log of Mountains B	1.00	0.97
	Discontinuous Territory A	2.36**	2.49**
	Discontinuous Territory B	1.00	0.96
	New State A	1.36	1.37
	New State B	0.94	0.93
	Instability A	1.46*	1.32
	Instability B	1.14	1.07
	Ethnic Fractionalization A	0.80	0.59
	Ethnic Fractionalization B	1.09	1.00
	Intercept	0.04***	0.07***
	Model χ^2	25.79***	22.46**
	Observations	7,365	7,365

Table 1: Logit regressions of onsets and incidence of interstate conflict. Mismatch constructed as absolute value of (Military Ratio – GDP Ratio). Cell entries are odds ratios. Robust standard errors clustered by countries' dyad. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

acteristics of the traditional bargaining break down approach: the status quo does matter in our theory; generally even the payoffs conditional on war do depend on the initial p because it is difficult to alter all power relations in one shot; and the outside option utility does indeed depend also on p, precisely because even the future mismatch after a war depends in expectation also on p. Moreover, we can show that the role of the mismatch continues to be critical even in the extreme case in which the winner of a war is assumed to be able to choose freely, without any friction, the new distribution of political power.

Understanding that, in reality, the mismatch between powers is more important than the balance of power debate focusing on a one dimensional notion of power should be important not only in international and national relations, but also potentially in other subfields of political economic studies. In future research, we will therefore propose the framework of this paper also to study social and family conflicts, politics/bureaucracy dynamics, and even institutional design, given that no existing theory of institutional design takes into account that the optimal set of institutions may depend on the current distribution of the other types of power that do not depend on the institutions being chosen.

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A Proofs

A.1 Proof of Lemma 1

Assume $m_t > p_t$. We need to show that $m_t + a_t > p_t + b_t$ and that $m_t - a_t > p_t - b_t$. We first show that $m_t + a_t > p_t + b_t$.

$$\begin{aligned} m_t + a_t &> p_t + b_t \\ m_t + g\left(1/2 - |m_t - 1/2|\right) &> p_t + g\left(1/2 - |p_t - 1/2|\right) \\ (m_t - p_t) &> g\left(|m_t - 1/2| - |p_t - 1/2|\right) \end{aligned}$$

There are three cases: (a) $1/2 > m_t > p_t$, (b) $m_t > p_t > 1/2$, (c) $m_t > 1/2 > p_t$.

In case (a), we have:

$$\begin{aligned} (m_t - p_t) &> g\left((1/2 - m_t) - (1/2 - p_t)\right) \\ (m_t - p_t) &> g\left(p_t - m_t\right) \end{aligned}$$

which is satisfied, since $m_t > p_t$ and $g > 0$.

In case (b), we have:

$$\begin{aligned} (m_t - p_t) &> g\left((m_t - 1/2) - (p_t - 1/2)\right) \\ (m_t - p_t) &> g\left(m_t - p_t\right) \end{aligned}$$

which is satisfied, since $g < 1$.

In case (c), we have:

$$\begin{aligned} (m_t - p_t) &> g\left((m_t - 1/2) - (1/2 - p_t)\right) \\ (m_t - p_t) &> g\left(m_t - (1 - p_t)\right) \end{aligned}$$

which is satisfied, since $(1 - p_t) > p_t$ and $g < 1$.

We now show that $m_t - a_t > p_t - b_t$.

$$\begin{aligned} m_t - a_t &> p_t - b_t \\ m_t - g\,(1/2 - |m_t - 1/2|) &> p_t - g\,(1/2 - |p_t - 1/2|) \\ (m_t - p_t) &> g\,(|p_t - 1/2| - |m_t - 1/2|) \end{aligned}$$

There are three cases: (a) $1/2 > m_t > p_t$, (b) $m_t > p_t > 1/2$, (c) $m_t > 1/2 > p_t$.

In case (a), we have:

$$\begin{aligned} (m_t - p_t) &> g\,((1/2 - p_t) - (1/2 - m_t)) \\ (m_t - p_t) &> g\,(m_t - p_t) \end{aligned}$$

which is satisfied, since $g < 1$.

In case (b), we have:

$$\begin{aligned} (m_t - p_t) &> g\,((p_t - 1/2) - (m_t - 1/2)) \\ (m_t - p_t) &> g\,(p_t - m_t) \end{aligned}$$

which is satisfied, since $m_t > p_t$ and $g > 0$.

In case (c), we have:

$$\begin{aligned} (m_t - p_t) &> g\,((1/2 - p_t) - (m_t - 1/2)) \\ (m_t - p_t) &> g\,((1 - m_t) - p_t) \end{aligned}$$

which is satisfied, since $(1 - m_t) < m_t$ and $g < 1$. ■

A.2 Proof of Proposition 2

As shown in the text, the cost thresholds below which A prefers to wage war is

$$\bar{c}_A = M_1 + \delta\left[(2m_1 - 1)(b_1 + M_1\Omega_1) + \frac{(\Omega_1)^2}{2}\right] \tag{3}$$

First, we want to show that A's dynamic incentive to attack is positive (and, thus, the ex-ante probability it attacks grows with δ) if $m_1 > m^* \in (1/4, 1/2)$ and negative otherwise.

A's dynamic incentive to attack is given by:

$$\frac{\partial \bar{c}_A}{\partial \delta} = (2m_1 - 1)\, b_1 + (2m_1 - 1)(M_1\Omega_1) + \frac{(\Omega_1)^2}{2} \tag{4}$$

When $m_1 \geq 1/2$, the first and second terms in equation (4) are weakly positive and the third term is strictly positive. Thus, A's dynamic incentive is strictly positive.

When $1/2 \geq m_1 > p_1$, we have $a_1 = gm_1$, $b_1 = gp_1$, $\Omega_1 = g(m_1 - p_1)$.

Thus, A's dynamic incentive becomes:

$$\frac{\partial \bar{c}_A}{\partial \delta} = (2m_1 - 1)(gp_1) + (2m_1 - 1)g(m_1 - p_1)^2 + \frac{g(m_1 - p_1)^2}{2} \tag{5}$$

When $m_1 = 1/2$, we have $\frac{\partial \bar{c}_A}{\partial \delta} = \frac{g(m_1 - p_1)^2}{2}$, which is strictly positive.

When $m_1 < 1/2$, the first term is negative. When is the sum of the second and the third term positive? We have:

$$\begin{aligned}
\frac{g^2(m_1 - p_1)^2}{2} + (2m_1 - 1)g(m_1 - p_1)^2 &> 0 \\
\frac{g^2(m_1 - p_1)^2}{2} &> (1 - 2m_1)g(m_1 - p_1)^2 \\
\frac{g}{2} &> 1 - 2m_1 \\
m_1 &> \frac{2 - g}{4}
\end{aligned}$$

This means that, when $m_1 < \frac{2-g}{4} \in \left(\frac{1}{4}, \frac{1}{2}\right)$, A's dynamic incentive is negative. When $1/2 > m_1 > \frac{2-g}{4} \in \left(\frac{1}{4}, \frac{1}{2}\right)$, A's dynamic incentive is the sum of a negative term plus a positive term. A's dynamic incentive is positive as long as the latter dominates the former. Since A's dynamic incentive is continuous in m_1 and it is strictly positive for $m_1 = 1/2$, it will continue to be positive for values of $m_1 < 1/2$ in a neighborhood of $1/2$.

Second, we want to show that, when A is sufficiently forward looking and sufficiently military disadvantaged, the dynamic incentive dominates the static one and A does not have any incentive to attack.

Does A ever lacks an incentive to attack? The static incentive to attack is positive. The only case where A can lack any incentive to attack is when the dynamic incentive is sufficiently negative and the discount factor is sufficiently high. Since A's dynamic incentive is positive for $m_1 > 1/2$, consider the case where $m_1 < 1/2$ and $\delta = 1$. A lacks any incentive to attack if:

$$
\begin{aligned}
M_1 + \delta(2m_1 - 1)b_1 + \delta\left(\frac{(\Omega_1)^2}{2} + (2m_1 - 1)M_1\Omega_1\right) &< 0 \\
M_1 + (2m_1 - 1)gp_1 + \left(g\frac{(M_1)^2}{2} + (2m_1 - 1)g\,(M_1)^2\right) &< 0 \\
M_1 + (2m_1 - 1)gp_1 + \left(2\left(m_1 - \frac{1}{4}\right)g\,(M_1)^2\right) &< 0 \\
M_1 + (2m_1 - 1)gp_1 + 2\left(m_1 - \frac{1}{4}\right)g\,(M_1)^2 &< 0 \\
M_1 + 2g\left(m_1 - \frac{1}{2}\right)p_1 + 2g\left(m_1 - \frac{1}{4}\right)(M_1)^2 &< 0 \\
\frac{m_1 - p_1}{2g} < \left(\frac{1}{2} - m_1\right)p_1 + \left(\frac{1}{4} - m_1\right)&(m_1 - p_1)^2
\end{aligned}
$$

Consider $p_1 \in (0, 1/4)$. As m_1 converges to p_1 from above, the LHS converges to 0, while the RHS converges to $(1/2 - p_1)p_1 > 0$. This shows that, for m_1 sufficiently low, there are initial power states (m_1, p_1) with $m_1 > p_1$ such that a forward looking politically under-represented player lacks any incentive to attack the politically over-represented player. Note that this is not true when players only care about the current distribution of resources.

A.3 Proof of Proposition 3

As shown in the text, the cost thresholds below which B prefers to wage war is:

$$\overline{c}_B = -M_1 + \delta \left[-(2m_1 - 1)(b_1 + 3M_1\Omega_1) - 3\frac{(\Omega_1)^2}{2} \right] \tag{6}$$

First, we want to show that B's dynamic incentive to attack is positive if $m_1 < m^{**} \in (\frac{2-g}{4}, \frac{1}{2})$ and negative otherwise.

B's dynamic incentive to attack is given by:

$$\frac{\partial \overline{c}_B}{\partial \delta} = -(2m_1 - 1)(b_1 + 3M_1\Omega_1) - 3\frac{(\Omega_1)^2}{2} \tag{7}$$

When $m \geq 1/2$, B's dynamic incentive to attack is negative. When $1/2 > m_1 \geq p_1$, we have $a_1 = gm_1$, $b_1 = gp_1$, $\Omega_1 = g(m_1 - p_1)$. In this case, B's dynamic incentive to attack is positive if:

$$\begin{aligned}
-(2m_1 - 1)(b_1 + 3M_1\Omega_1) - 3\frac{(\Omega_1)^2}{2} &> 0 \\
-(2m_1 - 1)\, g \left(p_1 + 3(M_1)^2\right) - 3g^2\frac{(M_1)^2}{2} &> 0 \\
(1 - 2m_1)\left(p_1 + 3(M_1)^2\right) &> 3g\frac{(M_1)^2}{2} \\
(1 - 2m_1)\, p_1 + 3\left(1 - 2m_1 - \frac{g}{2}\right)(M_1)^2 &> 0
\end{aligned}$$

A sufficient condition for the inequality above to hold is $\left(1 - 2m_1 - \frac{g}{2}\right) > 0$, or $\frac{2-g}{4} < m_1$. When $m_1 = \frac{2-g}{4}$, B's dynamic incentive is strictly positive and equal to $(1 - 2m_1)p_1$. Since B's dynamic incentive to attack is continuous in m_1, it will be positive also for $m_1 > \frac{2-g}{4}$ in a neighborhood of $m_1 = \frac{2-g}{4}$.

Second, we want to show that, when B is sufficiently forward looking and sufficiently military advantaged, the dynamic incentive dominates the static one and B has an incentive to attack.

Since B's dynamic incentive to attack is negative for $m_1 \geq 1/2$, consider $m_1 < 1/2$ and $\delta = 1$. We have:

$$\bar{c}_B = -M_1 + \delta \left[-(2m_1 - 1)(b_1 + 3M_1\Omega_1) - 3\frac{(\Omega_1)^2}{2} \right] > 0$$
$$(1 - 2m_1)\left(gp_1 + 3(M_1)^2\right) > M_1 + 3g\frac{(M_1)^2}{2}$$

Fix $p_1 \in (0, 1/2)$. As m_1 converges to p_1, the RHS converges to 0 and the LHS converges to $(1 - 2p_1)gp_1 > 0$. Thus, for δ sufficiently high and m_1 sufficiently high, there are power states (m_1, p_1) such that B will attack A for some realization of the cost of war, c_1. This does not happen in the static case.

A.4 Proof of Proposition 4

To show the first part of the proposition, assume $M_1 = m_1 - p_1 = 0$. When this is the case, $|1/2 - m_1| = |1/2 - p_1|$ and, thus, $\Omega_1 = 0$. The cost threshold become

$$\bar{c}_A = \delta(2m_1 - 1)gb_1 \quad (8)$$
$$\bar{c}_B = -\delta(2m_1 - 1)gb_1 \quad (9)$$

where $b_1 > 0$, since $p_1 \in (0, 1)$ and $g \in (0, 1)$. When $m_1 > 1/2$, there is a positive probability A attacks. When $m < 1/2$, there is a positive probability B attacks. Only when military power is perfectly balanced, that is, $m_1 = 1/2$, there is no chance of war.

Regarding the second and third part of the proposition, we first show that A's dynamic incentive is increasing in the mismatch for all (m_1, p_1) pairs with the exception of pairs with $m_1 in (1/2, 5/6)$ and $p_1 \in (0, 1/6)$. Since the static incentive is increasing in the mismatch, this implies that the overall incentive is increasing in the mismatch. Then, we show that, even when the dynamic incentive is not increasing in the mismatch, A's overall incentive (that is, $\bar{c}_A$) is increasing in the mismatch.

There are three cases to consider. Fix $p_1 \in (0, 1)$. How does the dynamic incentive change as m_1 grows (i.e., as the powers' mismatch, $M_1 = m_1 - p_1$, grow)?

Case 1: $m_1 \geq p_1 > 1/2$

$$\begin{aligned} b_1 &= g\left(1/2 - |p_1 - 1/2|\right) = g\left(1 - p_1\right) \\ \Omega_1 &= g\left(1/2 - |m_1 - 1/2|\right) - g\left(1/2 - |p_1 - 1/2|\right) = g(p_1 - m_1) \end{aligned}$$

Thus, A's dynamic incentive becomes

$$\frac{\partial \bar{c}_A}{\partial \delta} = (2m_1 - 1)g(1 - p_1) + g\left(2m_1 - \frac{1}{2}\right)(p_1 - m_1)^2 \tag{10}$$

We want to show that the partial derivative of the RHS with respect to m_1 is positive, i.e.,

$$\frac{\partial^2 \bar{c}_A}{\partial \delta \partial m_1} = 2g(1 - p_1) + 2g(p_1 - m_1)^2 - 2g\left(2m_1 - \frac{1}{2}\right)(p_1 - m_1) > 0 \tag{11}$$

This inequality is always satisfied since $\left(2m_1 - \frac{1}{2}\right) > 0$ and $(p_1 - m_1) < 0$.

Case 2: $m_1 > 1/2 \geq p_1$

$$\begin{aligned} b_1 &= gp_1 \\ \Omega_1 &= g(1 - m_1 - p_1) \end{aligned}$$

Thus, A's dynamic incentive becomes

$$\frac{\partial \overline{c}_A}{\partial \delta} = (2m_1 - 1)gp_1 + g\left(2m_1 - \frac{1}{2}\right)(1 - m_1 - p_1)^2 \tag{12}$$

We want to show that the partial derivative of the RHS with respect to m_1 is positive, i.e.,

$$\frac{\partial^2 \overline{c}_A}{\partial \delta \partial m_1} = 2gp_1 + 2g(1 - m_1 - p_1)^2 - 2g\left(2m_1 - \frac{1}{2}\right)(1 - m_1 - p_1) > 0 \tag{13}$$

This inequality is satisfied if $m_1 > (1 - p_1)$.

When $(1 - p_1) > m_1 > 1/2$, the inequality is satisfied if:

$$p_1 + (1 - m_1 - p_1)^2 - \left(2m_1 - \frac{1}{2}\right)(1 - m_1 - p_1) > 0 \tag{14}$$

$$(1 - m_1 - p_1)^2 > \left(2m_1 - \frac{1}{2}\right)(1 - m_1 - p_1) - p_1 \tag{15}$$

$$(1 - m_1 - p_1) > \left(2m_1 - \frac{1}{2}\right) - \frac{p_1}{(1 - m_1 - p_1)} \tag{16}$$

$$(1 - p_1) > m_1 + \left(2m_1 - \frac{1}{2}\right) - \frac{p_1}{(1 - m_1 - p_1)} \tag{17}$$

Since we are considering the case $(1 - p_1) > m_1$, this inequality is satisfied for any power state (m_1, p_1) in this case if:

$$\left(2m_1 - \frac{1}{2}\right) - \frac{p_1}{(1 - m_1 - p_1)} < 0 \tag{18}$$

$$\left(2m_1 - \frac{1}{2}\right) < \frac{p_1}{(1 - m_1 - p_1)} \tag{19}$$

$$(1 - m_1 - p_1)\left(2m_1 - \frac{1}{2}\right) < p_1 \tag{20}$$

$$\frac{5}{2}m_1 - 2(m_1)^2 - 2m_1p_1 - \frac{1}{2} + \frac{p_1}{2} < p_1 \tag{21}$$

$$\frac{5}{2}m_1 - 2(m_1)^2 - 2m_1p_1 - \frac{1}{2} < \frac{p_1}{2} \tag{22}$$

$$\frac{\left(\frac{5}{2}m_1 - 2(m_1)^2 - \frac{1}{2}\right)}{\left(\frac{1}{2} + 2m_1\right)} < p_1 \tag{23}$$

When $m \in (1/2, 1)$, the LHS maximum value is $\left(\frac{7}{4} - \frac{\sqrt{10}}{2}\right) \approx \frac{1}{6}$ Thus, the inequality is satisfied for any (m_1, p_1) pair such that $m_1 > 1/2 \geq p_1$ if $m_1 > 1 - p_1$ or if $m_1 < 1 - p_1$ and $p_1 > 1/6$.

Case 3: $1/2 \geq m_1 > p_1$

$$\begin{aligned} b_1 &= gp_1 \\ \Omega_1 &= g(m_1 - p_1) \end{aligned}$$

Thus, A's dynamic incentive becomes

$$\frac{\partial \overline{c}_A}{\partial \delta} = (2m_1 - 1)(gp_1) + g\left(2m_1 - \frac{1}{2}\right)(m_1 - p_1)^2 \tag{24}$$

which is strictly increasing in m_1.

To complete the proof, we want to show that, in Case 2 ($m_1 > 1/2 \geq p_1$), the ex-ante probability of war (that is, the sum of the static and dynamic incentive) is increasing in m_1 even when the dynamic incentive alone is not. We have:

$$\frac{\partial \overline{c}_A}{\partial m_1} = 1 + \delta\left[2gp_1 + 2g(1 - m_1 - p_1)^2 - 2g\left(2m_1 - \frac{1}{2}\right)(1 - m_1 - p_1)\right] \tag{25}$$

This inequality is positive if:

$$\frac{1}{2g} > \delta\left[-p_1 - (1 - m_1 - p_1)^2 + \left(2m_1 - \frac{1}{2}\right)(1 - m_1 - p_1)\right] \tag{26}$$

When $m_1 > (1 - p_1)$, the RHS is negative and the inequality is always satisfied (since the LHS is positive). When $m_1 < (1 - p_1)$, the inequality is satisfied for any $\delta \in [0, 1]$ and any $g \in (0, 1)$ if:

$$\frac{1}{2} > -p_1 - (1 - m_1 - p_1)^2 + \left(2m_1 - \frac{1}{2}\right)(1 - m_1 - p_1) \tag{27}$$

$$\frac{1}{2} > \frac{3p_1}{2} - 3\,(m_1)^2 - 4m_1p_1 - (p_1)^2 + \frac{9m_1}{2} - \frac{3}{2} \tag{28}$$

For $m_1 \in (1/2, 1)$ and $p_1 \in (0, 1/2]$, the maximum value the RHS can take is $3/16$ (when $p_1 = 0$ and $m_1 = 3/4$), which is less than $1/2$.

A.5 Proof of Proposition 5

Following a first war, a second war is more likely if the mismatch increases, that is, when $M_2 > M_1$. $\Omega_t \geq 0$ if and only if $\pi_t \geq \mu_t$, that is, if and only if political power is more unbalanced than military power and, thus, by our assumptions on the technology of power shifts (Assumption 1), the change in political power after a war is smaller than the change in military power. When $\Omega_t > 0$ ($\Omega_t < 0$), war increases the mismatch if the politically under-represented player wins (loses) and reduces it otherwise. Thus, if $\pi_t \geq \mu_t$, then $\Omega_t \geq 0$ and a second war is more likely if the politically under-represented player wins. When, instead, $\pi_t < \mu_t$ we have $\Omega_t < 0$ and a second war is more likely if the politically over-represented player wins.

A.6 Proof of Proposition 6

The (expected) mismatch change is:

$$E[M_2] - M_1 = [m_1(M_1 + \Omega_1) + (1 - m_1)(M_1 - \Omega_1)] - M_1 = (2m_1 - 1)\,\Omega_1$$

where:

$$\Omega_1 = g\left[(\pi_1)^2 - (\mu_1)^2\right]$$

The expected mismatch decreases — that is, $E[M_2] - M_1 < 0$ — if $m_1 < 1/2$ and $\pi_1 > \mu_1$ or if $m_1 > 1/2$ and $\mu_1 > \pi_1$. On the other hand, the expected mismatch grows if $m_1 > 1/2$ and $\pi_1 > \mu_1$. (Note that the case $m_1 < 1/2$ and $\pi_1 < \mu_1$ does not exist when $m_1 > p_1$).