Identity-Based Elections

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August 15, 2022

Abstract

We study the electoral implications of motivated media choice by Bayesian citizens aiming to preserve their political identity. In addition to their chosen media, citizens are somewhat exposed to outside information, which they try to counteract. When the outside information is unbiased, substantial political advantage may accrue to the side whose base is less exposed to it, or if that base incorrectly believes that it is imprecise or biased. Biased outside information works against the side that propagandizes. Finally, propaganda is beneficial only if citizens are unaware of its bias or in the case of a regime with censorship.

JEL codes: D72, D83, D90
Keywords: Behavioral Voters, Belief-based Utility, Information Aggregation

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‡We thank conference and seminar participants at UC Berkeley, Caltech, Princeton, Harvard, Georgetown, Rutgers, SMU, Vanderbilt, University of Utah, LSE, LBS, PSE, QMUL, University of Venice, FGV - Rio de Janeiro, CREST, SITE 2022, Stony Brook GT Conference, NASMES 2022, Ashoka University, and the Delhi School of Economics for useful comments and suggestions. All mistakes are our own.
1 Introduction

“More often than not, citizens do not choose which party to support based on policy opinion; they alter their policy opinion according to which party they support.”

In the era of the Internet, people can now choose from a plethora of possible news sources. Although traditional mainstream news sources continue to exist, many new media sources have emerged in just the past two decades. Figure 1, which is borrowed from Benkler et al. (2017), maps how 1.25 million news articles from 25,000 outlets were shared on Twitter. This richness of the media landscape has allowed people to more precisely tailor their media choices to their wants. However, the incredible diversity of viewpoints on offer, combined with new technologies, has also facilitated the formation of “echo chambers” or “filter bubbles” which insulate people from possibly contrary perspectives offered by traditional media outlets.

Figure 1: Pattern of news sharing on Twitter

At the same time, trust in traditional media has declined markedly over the past two decades. This distrust has developed along radically different paths on each side of the political spectrum, particularly in the last five years in the United States. As can be seen
in figure 2 below, the difference in the level of trust in mainstream media between Repub-
licans and Democrats is noteworthy.

Indeed, as noted by the Pew Foundation in Jurkowitz et al. (2020), “one of the clear-
est differences between Americans on opposing sides of the political aisle is that large
portions of Democrats express trust in a far greater number of news sources.” For in-
stance, as noted in Benkler et al. (2017) and illustrated in figure 1 during the 2016 US
presidential campaign, “Pro-Clinton audiences were highly attentive to traditional me-
dia outlets, which continued to be the most prominent outlets across the public sphere,
alongside more left-oriented online sites. But pro-Trump audiences paid the majority of
their attention to polarized outlets that have developed recently, many of them only since
the 2008 election season. [...] Breitbart News became the center of a distinct right-wing
media ecosystem, surrounded by Fox News, the Daily Caller, the Gateway Pundit, the
Washington Examiner, Infowars, Conservative Treehouse, and Truthfeed.”

Figure 2: Asymmetry in trust in mass media

These two phenomena — the emergence of a dense array of media outlets and partisan
distrust of mainstream media — are likely affecting the formation and updating of polit-
cal beliefs, and as a consequence, may be influencing voter decisions. But can this new
information environment generate aggregate beliefs biased enough to swing an election?
In this study, we seek to shed light on this question.

Lee (2010) finds that trust in media is negatively correlated with conservatism and Republican-leaning
views. Pennycook & Rand (2019) note that Democrats trust mainstream news more than Republicans,
with the difference ranging between 11.5 and 14.7 percent. According to Jones (2004), “only 16.5 percent of
Democrats (including Democratic-leaning independents) can be classified as media skeptics compared with nearly 40
percent of Republicans and Republican-leaning independents.”
In the U.S., the influence of the above-mentioned phenomena on electoral outcomes is compounded by the particularly polarized landscape, in which traditional ideological, religious, and racial identities are being replaced by overlapping *meta-identities* that align almost entirely with the Democratic and Republican political camps. Citizens have become less responsive to new information and real national problems, as if political affiliations determine what information people absorb, rather than the other way around (see, for instance, *Mason* (2018) and *Kahan* (2017)).

Given this situation, we build a model in which political identities drive individual media choices. Media choice is modeled in an environment where the agent may also be exposed to information from outside her chosen set of media outlets. Our basic premise of a dichotomy of chosen and outside media outlets allows us to consider both liberal and illiberal democracies within the same framework. We assume an individual chooses specific media to follow, but is to some extent also exposed to the outside world, namely the broader media environment which she does not explicitly choose. We refer to media that individuals choose, in full awareness of their bias, as Inside media. However, individuals are also exposed to some extent to media they do not choose which we refer to as Outside media. The character of the Outside media is determined by the media landscape prevailing in a society.

By making various assumptions about the Outside media, we are able to tackle a variety of questions. In the context of a liberal democracy, such as the U.S., while individual outlets may have some political bias, the mainstream media, as a whole, is generally considered to be free. Thus, the Outside information that a citizen is involuntarily exposed to originates from a relatively unbiased source. Although the Outside media is conceptualized as being unbiased in liberal democracies, citizens may differ in their exposure to it, or in their beliefs regarding its bias or degree of precision. On the other hand, in an illiberal democracy, the media landscape might be more influenced by one political side, such that the structure that generates Outside information is biased. We consider environments in which this bias and the degree of precision is common knowledge, as well as those in which some or all citizens hold incorrect beliefs about the bias or precision of the Outside media.

The core setup of the model is decision-theoretic and fairly straightforward. Individuals receive two pieces of news: one from the exogenous Outside media and one from Inside media, i.e., media endogenously chosen by the citizen. The different levels of

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2It is hard to define mainstream media in the current rapidly evolving media environment. Broadly speaking, we use the term to refer to traditional corporate legacy outlets which employ standard fact-checking processes and try to responsibly synthesize and diffuse the best information available.
exposure to Outside information means that Outside information is, in our benchmark setup, noisier for some individuals and therefore affects their beliefs to a lesser extent. The In-media source is chosen in a self-serving behavioral way, as explained below. Individuals are fully rational in the way they process information and update their beliefs based on the two signals. They then choose between the candidates according to their posterior beliefs. In sum, the election aggregates all votes, each of which is based on two conditionally independent signals about the candidates.

The model’s key behavioral assumption does not concern information processing, but rather the preferences that drive the choice of In-media. Each choice of In-media is a particular known signal structure (under commitment). We assume that an individual identifies with a particular party, either on the Left or the Right, and wishes to preserve their political faith. They choose In-media to maximize the likelihood that after they consume all available information, they will believe the party they identify with is the better match for the state of the world. In other words, individuals make their media choice in order to shield themselves from news which is potentially unfavorable to their political identity.

In-media readership can be viewed as a long-term choice rather than one that is instrumental to voting in a specific election (such that the utility is not derived from the voting itself but rather purely from ex-post beliefs). Alternatively, the choice of Inside media can be thought of as being made instrumentally in the context of a particular election, in which case we can think of agents as deriving explicit utility from being able to vote sincerely for the party they are aligned with.

Equivalently, one can think of agents as having two selves — a heart and a mind. The heart chooses Inside media in order to preserve the individual’s political identity, whereas the mind processes all the information it receives in a rational manner and votes for the party it believes to be superior. Here, objective of the heart could alternatively be to persuade the mind to vote for the heart’s preferred party, while the mind simply prefers to vote for the correct party.

To highlight the electoral consequences of variation in exposure to mainstream media along partisan lines, we present our benchmark results for a setup in which the two sides are perfectly symmetric, except in the extent to which they are exposed to information from mainstream media.

An asymmetry in the exposure of agents to mainstream information (Out-media) implies that the type of media they actively choose (In-media) can differ radically, which can result in a substantial electoral advantage for the side less exposed to mainstream media. In the example below, we examine the extent of that advantage, assuming that
each citizen votes for the party she rationally believes to be superior. The main results are qualitatively unchanged even if we assume that an individual votes for the party she is culturally affiliated with only if she believes it is the superior party, and abstains otherwise. In this case, all winning margins will simply be halved.\footnote{We assume infinite population size, therefore being pivotal is not an concern for voters in our model.}

**Illustrative example:** Assume a symmetric benchmark in which an equal proportion of countably infinite voters have partisan affiliations on each side, referred to as left (L) and right (R). There are two equally likely states of the world ($\omega \in \{L, R\}$), differentiated by which of the two candidates is superior. The only asymmetry between the two sides is that the left is more exposed to Outside news than the right. Assume, for instance, that the left-wing individuals receive i.i.d. symmetric binary signals from the Outside media with precision $t_L = 0.75$, while right-wing individuals receive noisier Outside signals, with precision of only $t_R = 0.51$. As a baseline, we consider electoral outcomes in the absence of Inside media — that is, when the agent is only exposed to an Outside signal. The winning margin and winning probability for the R side are then as follows:

<table>
<thead>
<tr>
<th></th>
<th>Ex-Ante</th>
<th>$\omega = R$</th>
<th>$\omega = L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>R Win Margin</td>
<td>0%</td>
<td>+26%</td>
<td>-26%</td>
</tr>
<tr>
<td>R Win Prob</td>
<td>50%</td>
<td>100%</td>
<td>0%</td>
</tr>
</tbody>
</table>

Thus, asymmetric exposure to mainstream media generates symmetric electoral outcomes. In this baseline case, the superior candidate is always elected, i.e., information is perfectly aggregated. No personal media choice is made by citizens, and thus political faith, whether with R or L, plays no role.

Now suppose that individuals can also optimally curate Inside media sources. In this case, their voting decision is made after rationally updating their beliefs based on two signals, rather than one. If the media is chosen in order to maximize the chance preserving their political identity, then the outcome of the election is no longer symmetric. In fact, it may be drastically skewed. In this example, the winning margin and winning probability for the R side are as follows:

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>R Win Margin</td>
<td>+28%</td>
<td>+54%</td>
<td>+2%</td>
</tr>
<tr>
<td>R Win Prob</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>
In this case, the R side has an ex-ante winning margin advantage in the election, but, surprisingly, it also has an ex-post advantage. In other words, the R side wins the election in either state of the world in this example and information is not aggregated, despite the fact that individuals vote solely on the basis of the information they have received and used to rationally update their beliefs.

The key drivers of this vote aggregation result are two features of media choice. First, because media choice is based on preserving, as often as possible, one’s political identity, it is equivalent, in the aggregate, to maximizing the ex-ante winning margin of the party with which an individual identifies. However, maximizing the ex-ante winning margin is different from maximizing the probability that their preferred party will win. The latter is more closely related to the ex-post winning margin conditional on the realized state.

Second, in a certain parameter range, the media choices made by citizens on either side of the political spectrum differ not only quantitatively but also, and this is a crucial distinction, qualitatively. The choice of the side less-exposed to Outside media (the Right in our example) is a one-sided signal structure, in which news favorable to one candidate is frequently realized and thus not so informative, while unfavorable news is rare and hence very informative. This is analogous to a partisan media outlet, designed such that favorable news from it overwhelms unfavorable Outside news and persuasion success is independent of the realization of the Outside signal.

In contrast, the side more exposed to Outside media (the Left in our example) faces an Outside signal that is more informative and is thus more difficult to overwhelm. They therefore, choose media which is more balanced in that it provides a mix of favorable and unfavorable news in either state of the world. Here, the persuasion success depends also on the realization of the Outside signal, such that the agent can sometimes receive a favorable Outside signal which counteracts an unfavorable Inside signal and thereby preserves her political faith.

Importantly, in a world without a rich set of signal structures (In-media) to choose from — even if partisan biases still drive media choice — we would not see so stark an aggregate electoral bias as in the example above.

Failure of information aggregation occurs when the two sides are symmetric in every respect except that one side is moderately exposed to mainstream media while the other has little exposure to it. As we will show, party R would win regardless of the state of the world even if there were slightly more agents of type L than type R, or when the prior leans towards the L side. Our benchmark assumption of symmetric common priors stacks the cards against our results. If agents on both sides have heterogeneous priors biased towards their side, then the scope for failure of information aggregation is greater.
In the benchmark model, the bias is in the preferences and is non-cognitive, although the model can accommodate other cognitive behavioral assumptions. The results do not change for a misspecified model in which the R side is exposed to the same Outside media landscape as the L side, but the R side distrusts the mainstream media and believes that the Outside signal is noisier than it actually is.

Our modeling strategy, which is based on the dichotomy between Inside media and Outside media, can also be used to explore other important questions. We initially assume that the Outside media news is unbiased, but allowing it to be biased makes it possible to consider government propaganda.

In section 5 we therefore imagine a propaganda regime. Citizens know that the information from the official media is biased and pro-government. More specifically, the official media delivers an unbiased message only with some probability and a pro-government message otherwise. That is, the government-influenced media has limited commitment to the truthful reporting of unbiased news and this is common knowledge.

In a media censorship regime, the official media is the only media available. Information is aggregated correctly only if the bias is low. For a large enough bias, the propaganda works, and the government wins an election regardless of the state. Without censorship, on the other hand, citizens, whether they support the government or oppose it, can also obtain information from sources of their choice. If we assume that citizens make this choice in order to preserve their political identity, then the electoral outcome is starkly different. First, there is no region of the parameter space in which the propagandizing party wins in all states of the world. Second, if the government media bias is strong enough, then the opposing side can win regardless of the state of the world. Thus, propaganda backfires without censorship of non-governmental media. When there is a failure of information aggregation, it is in favor of the anti-government side. This reversal is striking and occurs because individuals from the two political camps choose qualitatively different Inside media. This provides a possible explanation for the restrictions on free media observed in authoritarian states.

The general setup can be adapted to explore salient misspecified versions of the model, in which some of the citizens also have a cognitive bias. If some citizens misperceive the actual distribution of the Out-Media signal, then their In-Media choice may be individually suboptimal. The case in which the official government Out-media signal is biased, but pro-government citizens (mistakenly) believe it is not, is of particular interest. As we will show, without censorship of non-governmental media, propaganda benefits the propagandizing side only if citizens are unaware of it and wrongly believe that the government media is unbiased.
Finally, the cognitive bias case in which the Out-media signal is unbiased, but citizens on one side (mistakenly) believe it is biased against their side, is also of particular interest since it may apply to some liberal democracies, such as the US. In this case, we show that the significant political advantage accrues to the side that (wrongly) believes that the mainstream media, as a whole, is biased against them. This is very similar to electoral outcomes that occur when both sides are exposed to the same unbiased Outside media landscape and one side incorrectly believes that the mainstream media is imprecise.

This similarity implies that the results of information misaggregation and the benefits accruing to the political side that distrusts the mainstream media are robust to our interpretation of distrust. Whether we interpret distrust as an incorrect belief regarding the mainstream media’s precision or bias, we obtain qualitatively similar results.

2 Related Literature

Information biases and politics. We consider the electoral implications of behavioral biases in information acquisition following in the tradition of [Levy & Razin (2015) and Ortoleva & Snowberg (2015)]. The source of bias in both of them is correlation neglect, according to which individuals underestimate the correlation between their information sources. Although we also use a simple decision-theoretic problem aggregated to derive expected electoral outcomes, the behavioral bias in our benchmark case is not cognitive but rather resides in the preferences that drive information acquisition. Specifically, agents rationally update using all the information they receive.

Belief-based utility. The tradition of models with agents deriving utility from their beliefs goes back to [Akerlof & Dickens (1982)] who incorporate beliefs explicitly in the decision maker’s utility function. In their framework, beliefs are a choice variable, whereas, in our case, beliefs are a stochastic outcome of choosing a particular signal structure. Though cast in a different context, our model has a similar flavor to the Köszegi (2006) model of overconfidence, in which agents are unbiased in their beliefs (since they start from a correct prior and update rationally) but end up with a systematic bias in their choice due to the bias in their information collection process. Intuitively, agents derive intrinsic utility from believing that “something” is the case (specifically, that they have superior ability in some task, as in Köszegi’s case, or that their party is superior, as in our case), and therefore

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4For a survey of the recent literature on the electoral outcomes of these and similar cognitive biases, see Levy & Razin (2019). This literature continues to grow (see, for example, Little et al. (2020) on motivated reasoning cognitive bias).
they tend to collect information that preserves those beliefs as often as possible. Bénabou & Tirole (2016) provide a survey of the main findings to emerge from economic models of motivated beliefs.

The objective of preserving one’s political identity is also consistent with findings in a large strand of the psychology literature. Sherman & Cohen (2006) note that biases in the assimilation of information can be generated from the motivation to maintain and protect political identity. Motivated reasoning as a way of preserving political identity is also noted by Kahan (2017) and Kahan & Braman (2006). Furthermore, the ability to explicitly deliberate does not mitigate this phenomenon according to Kahan (2012) who finds that cognitive reflection exacerbates ideologically motivated reasoning. Kaanders et al. (2022) conduct an experiment with active information sampling and find that individuals are more likely to choose information that allows them to preserve their beliefs, which is consistent with the setup of our model.

Related theory. Similar to T. J. Feddersen & Pesendorfer (1996) or T. Feddersen & Pesendorfer (1997), we are considering the aggregation of information held by differently-informed voters through an election. However, in our case, voting is non-strategic, voters’ objective is political identity preservation, and the rich media landscape allows voters to learn. Our setup allows us to study interesting and salient failures of motivated media choice and information aggregation in elections. Motivated reasoning to preserve political faith is similar in spirit to minimizing cognitive dissonance, the electoral implications of which are considered in Acharya et al. (2018).

We borrow the theoretical techniques of Bayesian persuasion and use them to study electoral outcomes. Our focus is on conditionally aggregating individual decisions for a large population of individuals. In order to do so, we adapt and distill results developed by Kolotilin (2018), who building on Kamenica & Gentzkow (2011), requires the sender to choose an information structure while being uncertain about the receiver’s type. This is similar to our agent’s problem of choosing an information structure while bracing for an Outside signal. Lipnowski & Mathevet (2018) consider an information design problem with a benevolent sender who chooses a signal structure for a receiver with psychological preferences, an approach with similarities to our model of information choice by agents who derive belief-based utility.

Caplin & Leahy (2001) also considers belief-dependent utility and builds a two-period model of portfolio choice with anticipatory beliefs included in the agent’s utility function. In Brunnermeier & Parker (2005), agents choose beliefs to maximize their lifetime well-being under the assumption that agents increase their utility by anticipating gains.
Media and politics. Our model assumes that citizens can choose their In-media from a dense distribution of sources, spanning all possible biases. In other words, media in this model are passive and non-strategic, and media consumption is demand-driven only. Motivated by the fast-changing media landscape, there is a burgeoning literature that examines media bias — albeit usually from the supply side — in which media behave strategically. Gitmez & Molavi (2022) consider the problem of a single strategic media source that seeks to persuade a heterogeneous population of receivers to support the policy it espouses. They find that polarization among the receivers can make the media source less biased. In Perego & Yuksel (2022), growing competition between information providers leads to news specialization, which in turn amplifies social disagreement — a result with a similar flavor to our results. Gentzkow & Shapiro (2006) find that media outlets slant their reports so as to conform to prior beliefs held by agents and also that media bias is reduced in the presence of Outside information. In our case, the presence of asymmetry in the precision of Outside information is one of the factors that can lead to failures of information aggregation. Gentzkow et al. (2021) find that belief divergence among agents seeking to learn a sequence of states can be the result of small ideological differences in their trust in information sources and their beliefs about the state of the world. A strategically behaving media that biases its news to induce greater viewership may increase polarization according to Bernhardt et al. (2008). Their results hold even though individuals are aware of the media bias and update rationally. In Mullainathan & Shleifer (2005), agents consume news from different outlets depending on their desire to confirm pre-existing biases and they find that media strategically offer slanted news — an effect exacerbated by greater competition among the media.

Finally, our voting results are consistent with several empirical studies such as DellaVigna & Kaplan (2007) and Martin & Yurukoglu (2017) which show that the introduction of new (and largely conservative) media outlets resulted in a persistent and significant increase in Republican vote share. We describe the workhorse model in the next section. Then, we apply it to liberal democracies. We examine illiberal democracies last.

6In a similar vein, Hu et al. (2019) consider the role of media intermediaries who may broadcast information widely or tailor it to individuals. They characterize the conditions under which personalization increases polarization and find that polarization reduces if voters have a higher attention cost.

7As shown by Angelucci & Prat (2021), belief in fake news, while present, does not have a systematic partisan bias.
3 Model

Here, we outline the general setup and the structure of the solution, which remain very similar for each variant of the model that we consider in subsequent sections.

3.1 Setup

There are countably infinite agents, who are either type-\(R\) or type-\(L\) in equal proportion, which corresponds to their political identities. There are two states of the world (\(R\) and \(L\)) and two political sides/parties (\(R\) and \(L\)). The state of the world indicates which party is superior. All agents share a common and symmetric prior regarding the state of the world (\(P[\omega = R] := w = 0.5\)).

An agent receives two signals about the state of the world: an Inside signal and an Outside signal. The Inside signal is generated by a chosen signal structure, while the Outside signal is generated by an exogenously specified signal structure. We interpret the Inside signal as being generated by the media that the agent chooses to consume. The agent does not choose the Outside signal structure, rather it is simply a feature of the agent’s media environment. The outside media environment is general for now such that the signal the agent is exposed to and must brace for may or may not be biased. Furthermore, the agent may be imperfectly exposed to the Outside signal, or she may, in general, hold incorrect beliefs about it.

The timing of the game is as follows. First, each agent chooses her Inside signal structure. All agents then receive Inside and Outside signals. In the next stage, agents form Bayesian posteriors and realize belief-based utility. Each agent gains utility from holding a posterior belief that her party is more likely the superior party, which is equivalent believing that the state of the world more likely matches her type. Finally, agents vote sincerely. Because there is a countable infinity of agents, none of them is ever pivotal. Voting is therefore, non-strategic. Moreover, the action of voting does not impact agents’ utility, which is based on their beliefs.

We can specify an equivalent model in which actions are strategic. Suppose that each agent has two selves, a heart (sender) and a mind (receiver). The sender-self is of type \(R\) or \(L\) and chooses the In-media signal structure. The receiver-self votes sincerely based on her updated beliefs. The receiver-self forms her beliefs rationally after observing both the Inside and Outside signals. The objective of a type-\(R\) (\(L\)) sender-self is to maximize the likelihood that the receiver-self votes for party \(R\) (\(L\)).

\[\text{As we show in subsection 4.3, the key results of the model are robust to biased and non-common priors.}\]
be thought of as another interpretation of the agent’s problem. In what follows, however, we examine the model as described in the previous paragraph, in which each agent has a single self with belief-based utility.

Because the game is symmetric, we specify the problem for a type-\(R\) agent, and the specification for a type-\(L\) agent is analogous. The utility function of a type \(R\) agent is:

\[
U_R = \begin{cases} 
1 & \text{if } \mathbb{P}[\omega = R | S, s] \geq 0.5 \\
0 & \text{otherwise}
\end{cases}
\]

where \(S\) is the signal generated by the Inside signal structure, while \(s\) is the signal generated by the Outside signal structure. For the agent, maximizing her expected utility is equivalent to maximizing the likelihood that she holds a posterior according to which she believes that her type more likely matches the state of the world. We refer to this form of self-serving bias as political faith preservation. The agent’s objective can, therefore, be more simply expressed as:

\[
\mathbb{E}[U_R] = \mathbb{P}[\mathbb{P}[\omega = R | S, s] \geq 0.5]
\]

The agent’s problem is to choose an Inside signal structure so as to maximize equation 1. The action set available to the agent is the set of Inside signal structures specified below. We assume that in addition to choosing the Inside signal structure, the agent also takes the non-strategic action of sincere voting.\(^9\) We can alternatively assume that the agent abstains if her beliefs do not allow her to vote sincerely, that is in line with her posterior beliefs, for her preferred party. Both cases produce the same qualitative results while the winning margins are simply halved in the latter case.

We are interested in electoral outcomes, particularly in situations where the incorrect party wins, i.e., party \(R\) wins in state \(L\) or vice versa. We call these situations information aggregation failures. No welfare loss results from information misaggregation because the agents in our model gain no utility from voting or from electoral outcomes. However, we can talk about information aggregation as individuals process all the information they receive in a Bayesian way, thus the beliefs they hold are coherent (at least in our core model, without cognitive biases).

\(^9\)In the case of a tie, she votes for her preferred party.
Outside signal structure. Each agent receives an Outside signal \((s \in \{r,l\})\) with distribution:

\[
\mathbb{P}[s = l|\omega = L] = k, \quad \mathbb{P}[s = r|\omega = R] = m
\]

where \(k \in [0,1]\) and \(m \in [0,1]\). The Outside signal can be modified by imposing structure on \(k\) and \(m\), with the goal of exploring a variety of environments. In the baseline model described in subsection 4.1 we consider an environment with unbiased Outside signal structures by setting \(k = m\), and capture asymmetric exposure to Outside media by assuming that agents of type \(R\) receive a less precise signal than agents of type \(L\). In subsection 4.2 we examine media distrust by supposing that agents of type \(R\) believe incorrectly that they receive a less precise signal than agents of type \(L\). We introduce propaganda in favor of party \(L\) in section 5 by imposing \(k > m\). In that context, we consider the implications of censorship, and of individuals believing incorrectly that the mainstream media is unbiased. Finally, we consider perceived propaganda in favor of party \(L\) in subsection 6.2 by setting \(k = m\), and assuming that type-\(R\) agents believe incorrectly that \(k > m\).

Inside signal structure. The Inside signal structure is a finite set of signals \(S = \{S_1, S_2, \ldots, S_n\}\) that are correlated with the state of the world. We impose no restrictions on the agent’s choice of the Inside signal structure except that it must abide by the Martingale property that the expected posterior must equal the prior. We interpret this choice of signal structure as the choice of which combination of news media to consume news from. The rich set of signal structures represents the rich media landscape in which voters currently gather information. Furthermore, although an agent can choose a news media (a signal structure), she cannot choose the programming (the realization of a signal). We assume that both the Inside signal and the Outside signal are realized independently for all agents, conditional on the state of the world. This is a reasonable assumption in a media-rich environment where different media choices might have the same signal structure, but different realizations of the signal. Under independent realization of signals, there is no aggregate uncertainty in outcomes. Introducing correlation is straightforward, and the results have a similar flavor, albeit with aggregate uncertainty about electoral outcomes.

3.2 Structure of the solution

We solve the model with a general Outside signal in appendix A and then apply the solutions to specific structures of the Outside signal. We describe the solution for a type-\(R\) agent, where the solution for a type-\(L\) agent is analogous. The solution of the agent’s
problem in this model applies the techniques developed by Kamenica & Gentzkow (2011) and Kolotilin (2018). Following these papers, it is straightforward to show that three signals combined in two possible Inside signal structures are sufficient to solve the agent’s problem. The purpose of providing the solution structure is to highlight the intuition and drivers of our main results which concern the aggregation of beliefs and the voting behavior of a countable infinity of agents.

The agent’s problem. The agent chooses the Inside signal structure to maximize the likelihood of political faith preservation subject to the constraint on the Inside signal structure which specifies that the expectation of her posterior belief be identical to her prior belief.

Following the intuition described in Kamenica & Gentzkow (2011), we first calculate the agent’s expected utility as a function of her interim priors, namely after she has received the In-media signal, and before she has received the Out-media signal. We denote this interim posterior as $P[\omega = R | S_i]$. We will then obtain the optimal signal structure choice of the agent as a function of the precision of her Outside signal.

Figure 3: Expected utility as a function of the interim posteriors

In figure 3, we plot a type-$R$ agent’s expected utility (which equals the likelihood of preserving her political faith) as a function of her interim posterior for an Outside signal structure such that $k = m = 0.75$. For values of the interim posterior between 0 and 0.25, regardless of the realization of the Outside signal, the agent is unable to preserve her political faith. If the agent has an interim posterior equal to 0.25, and if she receives a favorable Outside signal, then her posterior expectation that the state of the world is $R$
is equal to 0.5, and she is just able to preserve her political faith. For values of interim posteriors between 0.25 and 0.75, she can preserve her political faith if she receives a favorable Outside signal, \( s = r \). The likelihood that she receives a favorable Outside signal increases with her expectation that the state of the world is \( \omega = R \). Finally, if the agent’s interim posterior is at least 0.75, then she is able to preserve her political faith regardless of the Outside signal.

Sufficient set of signals. As we detail in appendix A, three interim posteriors are key to solving the agent’s problem. The first is an interim posterior such that the agent is just able to preserve her political faith if she receives an unfavorable Outside signal (\( P[\omega = R | S_i] = 0.75 \) in figure 3). We refer to an Inside signal that generates such an interim posterior as a Good (G) signal. The second is an interim posterior that allows the agent to just preserve her political faith only if she receives a favorable Outside signal (\( P[\omega = R | S_i] = 0.25 \) in figure 3). We refer to an Inside signal that generates such an interim posterior as a Bad (B) signal. The third is an interim posterior such that the agent is certain that the state does not match her preferred party (\( P[\omega = R | S_i] = 0 \)). We refer to an Inside signal that generates such an interim posterior as a Terrible (T) signal. Any signals other than the three described above — Good, Bad, and Terrible — would be suboptimal. Furthermore, they can be combined into two signal structures, GT or GB, that are sufficient to solve the agent’s problem.

In figure 4, we plot the agent’s expected utility as a function of her interim posterior after observing the In-media signal and before observing the Out-media signal.

**Figure 4: Expected utility as a function of interim posteriors**

![Figure 4](image)
Figure 4(a) considers the case in which the Out-media has a less precise signal, and the interim posterior that allows an agent to just preserve her political faith if she receives a favorable Outside signal is close to her prior (at $P[\omega = R|S_i] = 0.4$). Similarly, the agent preserves her political faith regardless of the realization of the Outside signal if her interim posterior is at least $P[\omega = R|S_i] = 0.6$. Figure 4(b) presents a more precise Outside signal structure, which implies that the two key interim posteriors described above are more spread out away from the prior of $w = 0.5$.

For lower values of precision, such as in figure 4(a), the concave closure of the expected utility function is such that it would be optimal for the agent to choose a signal structure that mixes between $G$ and $T$, namely a $GT$ signal structure. On the other hand, if the Outside signal is more precise, as it is in figure 4(b), the concave closure of the expected utility is such that the optimal signal structure is $GB$.

**Interpreting the signal structures.** Note that the $GT$ signal structure is one-sided, in the sense that in the favorable state of the world, only signal $G$ is realized. Therefore, on observing a signal $T$, the agent is certain that the state of the world is unfavorable. We interpret $GT$ as media with a strong political slant. Favorable news is routinely reported by these outlets and is not very informative for Bayesian agents. When these outlets report unfavorable news, it is highly informative for Bayesian agents and will convince them that the state of the world is not favorable to their side.

On the other hand, $GB$ is two-sided in that either $G$ or $B$ can be realized in either state of the world, according to a predetermined probability distribution. This is akin to the agent consuming less one-sided media since it provides her with a mix of positive and negative news in either state of the world. The probability or realization of each signal ($G$ or $B$) changes depending on the state. The fact that a negative signal is sent by these media in either state of the world makes it less informative allowing a critique not to be irredeemably bad. In this case, the Outside news can be crucial in the preservation of the individual’s political identity since a favorable Outside signal can counteract an unfavorable Inside signal. In contrast, with the more slanted one-sided In-media type, the preservation of political identity does not depend on the realization of Outside news.

Individuals who face less exposure to Outside information prefer a $GT$ signal structure because they find a negative Outside signal easy to counteract, and a positive Outside signal, not very informative. On the other hand, individuals who face a more precise

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10 We use the definition of concave closure found in Kamenica & Gentzkow (2011).
11 For instance, media sources like Breitbart News for Republicans or the Huffington Post for Democrats.
12 We interpret the $GB$ signal structure to be media sources like the Wall Street Journal for Republicans or the New York Times for Democrats.
Outside signal prefer a GB signal structure because it allows them to sometimes use a favorable Outside signal realization in preserving their political identity.

Robustness checks. In Appendix A.3, we show that the nature of this problem and the optimal signal structures are robust to a small amount of uncertainty or noise in the threshold of belief required for political faith preservation. This kind of robustness is also noted in Kamenica & Gentzkow (2011) whose key results and structure of the solution hold in the presence of noise which makes the receiver’s belief threshold for choosing the sender-preferred action stochastic.

In appendix A.4, we show that all the results are robust to an additive function where the agent has a relatively small gain from voting for the correct party. This is because for low values of gain from being correct, the set of signal structures that solve the agent’s problem remain the same as in the benchmark. If there is a region of information aggregation failure present in the benchmark, then, although that region might shrink in size, it does not disappear when we include a small gain from voting for the correct party.

We further show in appendix A.5 that all the results remain unchanged if the agent gains linearly from holding posteriors favorable to her party, in addition to the gain from political faith preservation. This is because the choice of an Inside signal structure is invariant to the gain from favorable posteriors, and all electoral outcomes are determined by that choice. This robustness holds for all levels of that gain in the modified utility function.

We can see from these checks that the results are robust to small changes to the shape of the utility function through noise, or preferences for being correct or for holding more favorable beliefs. The key driver of the results is a sharp change in the agent’s utility when her beliefs cross a predetermined threshold.

In the following applications of the model, we consider various structures of $k$ and $m$ that admit specific interpretations and which can shed light on the electoral implications of politically motivated media choice.

4 Liberal democracy

In this section, we consider the case of a liberal democracy, in which the media is free, namely it is not beholden to any particular side. The mainstream media, as a whole, is unbiased and is perceived to be so by citizens. We later consider an illiberal democracy in which the media, as a whole, is influenced by one political camp.
4.1 Asymmetric exposure to mainstream media

This specification builds on the example presented in the introduction. Suppose that exposure to mainstream media differs systematically by party preference. In this case, there is an asymmetry in informational insularity between individuals with different political affiliations.

Individuals in this environment have access to a wide array of news and opinion outlets that allow them to consume a specific diet of chosen media. We are interested in their choice of In-media as a function of their exposure to Out-media and its effect on electoral outcomes. In this section, both agents receive unbiased Outside signals that are not fully informative, and we suppose that agents of type $R$ receive noisier Outside signals than agents of type $L$. The reverse case yields analogous results.

One way to interpret precision and exposure in the context of the Outside signal is as attention. Attention, or lack thereof, might be a feature of the media landscape or one’s social circle which determines the intensity, frequency, or clarity with which agents receive the signal from outside their chosen media diet. For instance, in the U.S., an asymmetry in information insularity between the two political parties may be due to the rural-urban sorting between Republicans and Democrats. An alternative interpretation of attention would be that signal precision and exposure capture the openness of an agent to receiving a signal from outside her chosen In-media. This openness, or lack thereof, may be due to the agent’s preference or beliefs regarding the trustworthiness of the Outside information, which may cause the agent to actively avoid exposure to mainstream media. The asymmetry in exposure can, therefore, be thought of as reflecting either the media landscape, or the agent’s preferences or beliefs, or some combination thereof.\footnote{Note that in this subsection, type-$R$ agents receive a less precise Outside signal than type-$L$ agents. In subsection 4.2, we consider a variant of the model in which type-$R$ agents receive the same precision of the Outside signal as type-$L$ agents, but they incorrectly believe that the Outside signal is less precise.}

The Out-media signal has the following structure

$$\mathbb{P}[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2}, \quad \mathbb{P}[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2}$$

where $t \in [0.5, 1]$ is the precision of the Outside signal and $\tau$ is the extent of exposure to that signal. We model the asymmetry in exposure by supposing $\tau = 1$ for agents of type $L$ and $\tau \in [0, 1]$ for agents of type $R$. An alternative isomorphic setup would simply specify different levels of precision for the Outside signal received by the two types of agents, such that the Outside signal’s precision for an agent of type $R$ is lower than that for an agent of type $L$.\footnote{Note that in this subsection, type-$R$ agents receive a less precise Outside signal than type-$L$ agents. In subsection 4.2, we consider a variant of the model in which type-$R$ agents receive the same precision of the Outside signal as type-$L$ agents, but they incorrectly believe that the Outside signal is less precise.}
**The example.** In the example appearing in the introduction, we set $t_R = 0.51$ and $t_L = 0.75$, which is equivalent to setting $t = 0.75$ and $\tau = 0.04$.\footnote{This value of $\tau = 0.04$ is only relevant for type-$R$ agents since we have set $\tau = 1$ for type-$L$ agents.} For these parameter values, we find that introducing the ability to choose one’s media transforms a fully symmetric election into one with failure of information aggregation, where party $R$ wins the election in both states of the world. This result is obtained despite agents being rational in their information processing.

<table>
<thead>
<tr>
<th>Parameters: $t = 0.75, \tau = 0.04$</th>
<th>Without In-media</th>
<th>With In-media</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\omega = R$</td>
<td>$\omega = L$</td>
</tr>
<tr>
<td>R Win Margin</td>
<td>0%</td>
<td>+26%</td>
</tr>
<tr>
<td>R Win Prob</td>
<td>50%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 1 shows that without In-media, asymmetric exposure to mainstream media does not provide the $R$-side with any advantage in winning margin or winning probability. The correct side wins in both states of the world. When the state of the world is $R$, then 51% of type-$R$ agents and 75% of type $L$ agents receive an Outside signal indicating that the state of the world is more likely to be $R$, and they vote for party $R$, which implies that the $R$-side wins with a margin of 26%. Because the Outside signal is unbiased, party $L$ wins with the same margin in state $L$.

**Mechanism.** The introduction of the Inside signal in the second row of table 1 results in an information aggregation failure that allows party $R$ to win regardless of the state of the world. This result is not ex-ante obvious.

Figure 5(a) plots the expected utility (which is equivalent to the likelihood of political faith preservation) for an agent choosing $GT$ and $GB$ signal structures as a function of the perceived precision of the Outside signal ($t' = \tau t + \frac{(1-\tau)}{2}$). When $t'$ is low, the agent requires very little to counteract an unfavorable Outside signal. At the same time, a favorable Outside signal is not very informative either.

If the agent chooses a $GT$ signal structure, she may receive a $G$ signal that allows her to preserve her political faith regardless of the realization of the Outside signal, or a $T$ signal which ensures that she does not preserve her political faith. The probability of realization of the $G$ signal equals 1 if the perceived precision of the Outside signal equals 0.5, resulting in the sure preservation of the agent’s political faith. As $t'$ increases, the $G$ signal must be realized less often in order to be more informative and thereby preserve
the agent’s political faith when she receives an unfavorable Outside signal. This reduces
the agent’s expected utility. When the Outside signal is perfectly informative, the agent
preserves her political faith only when her party is the superior one, which happens when
her type matches the state of the world.

In the agent chooses a GB signal structure, then the G signal is less likely to be realized
than if she had chosen a GT signal structure. This is because the B signal is more favorable
to the agent than a T signal and the Inside signal structure must satisfy the Martingale property.\textsuperscript{15} When \(t'\) equals 0.5, the Inside signals, G and B, are equally likely to be realized
in either state of the world and the agent’s expected utility is 0.75. She preserves her
political faith for sure when the signal G is realized, and with a probability 0.5 if B is
realized. As \(t'\) increases, the agent is able to preserve her political faith less often.

We see that a GT signal structure results in higher utility if the perceived precision
of the Outside signal is sufficiently low. For higher values of perceived precision, a GB
signal is optimal. The intuition behind this graph follows directly from figures 4(a) and
4(b). When the perceived precision of the Outside signal is lower, it is easier to counteract
an unfavorable Outside signal and a GT signal structure is better. For higher levels of
precision of the Outside signal, a GB signal structure is better for the agent because it
allows her to rely a favorable realization of the Outside signal to preserve her political
faith by overwhelming even an unfavorable realization of her Inside signal. Under the
parameter values described above \((t = 0.75 \text{ and } \tau = 0.04)\), a type-R agent chooses a GT
signal structure, whereas a type-L agent chooses a GB signal structure.\textsuperscript{16}

Figure 5(b) illustrates the probability of political faith preservation, which equals the
agent’s expected utility, ex-ante and conditional on the state of the world. The red lines
correspond to a type-R agent’s expected utility ex-ante (solid), in state R (dotted), and
in state L (dashed) when the agent chooses a GT signal structure. The type-R agent’s
expected utility is similarly represented in blue when she chooses a GB signal structure.
The graph is identical for type-L individuals, except that the favorable and unfavorable
states of the world are reversed.

In the favorable state, a GT signal structure allows an agent to preserve her political
faith with certainty because the agent receives the Good signal. For instance, a type-R
agent would receive an expected utility equal to 1 in state R if she chooses a GT signal
structure, which is illustrated by the dotted red line. Relative to a GT signal, a GB signal

\textsuperscript{15}Recall that a B Inside signal allows the agent to just preserve her political faith if she receives a favorable
Outside signal.

\textsuperscript{16}In Appendix B.1, we show that the GT signal structure is optimal for type-R agents if and only if
\(\tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq \frac{1}{\sqrt{2}}\). Because type-L agents receive an Out-media signal with a perceived precision \(t\),
they choose a GT signal structure for all values of \(t \leq \frac{1}{\sqrt{2}}\) and a GB signal otherwise.
structure allows the agent to preserve her political faith with a higher likelihood in the unfavorable state, but with a probability of less than 1 in the favorable state. We see this fact illustrated in the dotted and dashed blue lines in figure 5(b).

In the example, we considered parameter values such that the perceived precision for type-R agents is quite low, and so, they choose a GT signal structure. On the other hand, type-L agents have a greater perceived precision of the Outside signal, which induces them to choose a GB signal structure.

Given the parameter values in table 1, all type-R agents vote for party R when the state of the world is R. The dotted red line shows that the expected utility of an agent who chooses a GT signal structure equals 1 in the favorable state of the world. As denoted by the dashed blue line, a GB signal structure in the unfavorable state of the world implies that type-L agents preserve their political faith and vote for party L with a probability of 44%. Therefore, party R wins in state R.

When the state of the world is L, the expected utility of type-R agents is denoted by the red arrow on the dashed red line, while the expected utility of type-L agents is denoted by the blue arrow on the dotted blue line. For some parameter values, specifically when type-R agents perceive the Outside information to be very imprecise and type-L agents perceive it to be moderately precise, the red arrow is higher than the blue arrow — implying that even in state L, type-R agents preserve their political faith with a higher probability than type-L agents. Considering the parameter values in table 1, 96% of type-R agents vote for party R while 94% of type-L agents vote for party L. Therefore, party R wins even in state L. The range of parameters in which this kind of information mis-aggregation occurs is particularly salient because it corresponds to type-R agents being
relatively closed to Outside information while type-$L$ agents are relatively open, which could be caused by the asymmetry in trust in mainstream media discussed in the introduction.

**Discussion.** Every agent individually maximizes her likelihood of political faith preservation, but, for some parameters, this implies that party $L$ loses regardless of the state of the world.\footnote{On aggregate, if each agent maximizes her likelihood of political faith preservation, it is equivalent to each side ($L$ and $R$) maximizing their expected vote share.} If, instead, type-$L$ agents choose $GT$ as their Inside signal structure, then party $L$ can win in the correct state — implying that an ex-ante suboptimal individual choice can allow for ex-post optimal policy choice. In other words, party $L$ would benefit if it could convince its electorate to consume more politically slanted news. In the U.S. context, this would happen if the Democratic party notes that Republicans consume more slanted news, realizes that they could benefit electorally if Democrats also consume more slanted news, and successfully changes their news consumption patterns. We believe that, in line with the structure of the model, it is more likely that individual news consumption is driven by individual preferences, rather than by mandates from political parties, or by strategic considerations of which media members of the opposing political camp consume.

It is important to note that information aggregation failure occurs for low values of exposure ($\tau$) and intermediate values of Outside signal precision ($t$). If the Outside signal were less precise, for instance, if $t = 0.7$, then both types of agents would choose a $GT$ signal structure, and the correct party would always win. On the other hand, if the Outside signal were very precise, then too, the correct party would always win. If the state of the world is $L$, then compared to the outcomes in table\footnote{\ref{table:signal_types}} type-$L$ agents are more likely to receive a *Good* signal and vote for party $L$ in state $L$. Similarly, type-$R$ agents are more likely to receive a *Terrible* signal and vote for party $L$. Party $L$, therefore, receives more votes than party $R$ in state $L$.

In figure\ref{fig:signal_regions}(a), we consider all values of signal precision ($t$) and exposure ($\tau$). In the red-shaded area, both types of agents choose a $GT$ signal structure. In the blue-shaded area, both types of agents choose a $GB$ signal structure. In the purple-shaded region, type-$R$ agents choose a $GT$ signal structure, while type-$L$ agents choose a $GB$ signal structure. There is no region where type-$R$ agents choose a $GB$ signal structure and type-$L$ agents choose a $GT$ signal structure. As claimed in proposition\footnote{\ref{proposition:information_aggregation}} below, there is a region contained within the purple-shaded region where information aggregation fails, such that type-$R$ choose a $GT$ signal signal structure and type-$L$ agents choose a $GB$ signal struc-
Figure 6: Signal choices and results with asymmetric exposure

(a) Common priors
(b) Non-common priors: $w_R = 0.6, w_L = 0.4$

Proposition 1 In the environment specified in subsection 4.1, the correct candidate wins except in a region with intermediate Outside signal precision and low exposure among type-R agents. In the region of information misaggregation, type-R agents choose a GT signal structure and type-L agents choose a GB signal structure.

We show in appendix B.1 that the result in the above example is not knife-edge, and that the voting margins are continuous in $\tau$ and $t$ in the neighborhood of $\tau = 0.04$ and $t = 0.75$. In figure 6(a), we see that the light-purple-shaded area, which denotes the region of information aggregation failure, exists for low values of media exposure on the Right ($\tau$) and moderate values of Out-media precision ($t$).

When both types of agents choose a GT signal structure for their respective In-media and if the state of the world is $R$, then all type-R agents and some type-L agents vote for party $R$. If the state of the world is $L$, then all type-L agents and some type-R agents vote for party $L$. In this case, information aggregation failure is not possible. We show in appendix B.1 that the party that matches the state wins for all values of $\tau$ and $t$ such that a GB signal structure is optimal for both types. Furthermore, the parameter space is such that there is no situation in which type-R agents choose a GB signal structure and type-L agents choose a GB signal structure.

Party $R$ achieves an ex-ante winning margin advantage as a result of the lower exposure of type-R citizens. This a straightforward implication of the fact that it is easier for type-R citizens to preserve their political faith because they don’t need to brace for as
precise an Outside signal as type-L citizens. It is, however, striking that the political advantage for party $R$ can be so substantial for some parameters, that it can win regardless of the state of the world. Furthermore, while the region of information aggregation failure in figure 6(a) appears to be small, those parameter values may be particularly relevant in the case of the U.S. The intermediate values of Out-media precision at which information misaggregation takes place are high enough that type-L agents choose a two-sided $GB$ In-media signal structure, but not so high that party $L$ wins in state $L$. Furthermore, as will be shown in subsection 4.3 below, the parameter space in which information misaggregation occurs expands substantially when agents have non-common priors in favor of their preferred parties.

4.2 Distrust in mainstream media precision

We previously considered an environment in which type-$R$ agents receive a less precise Outside signal than type-$L$ agents, such that the model is correctly specified. We now show that the results remain unchanged if type-$R$ agents face an Outside signal structure that is identical to that faced by type-$L$ agents, but they incorrectly believe that the precision of the Outside signal is lower than it actually is. This misspecified model is particularly applicable to the case of the U.S. considering the asymmetry in trust in mainstream media between voters in the two main political camps (as discussed in the introduction). Crucially, note that Republicans have a lower level of trust in mainstream media than Democrats and that key Republican leaders have sought to amplify that distrust. In this case, we interpret distrust as reflecting an agents’ perception of the quality of mainstream media. In subsection 6.2, we interpret it as the perception of the bias of mainstream media.

The Outside signal has the following structure:

$$P[s = l|\omega = L] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2}, \quad P[s = r|\omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2}$$

where $t \in [0.5, 1]$ is the true precision of the Outside signal and $\tau$ is the extent of agents’ trust in the Outside signal. We specify the asymmetry in distrust by setting $\tau = 1$ for type-$L$ agents and $\tau \in [0, 1]$ for type-$R$ agents.

The choice of signal structures by either type of agent is identical to that in the previous specification. Again, we find there to be misaggregation of information within the region where type-$R$ agents choose a $GT$ signal structure and type-$L$ agents choose a $GB$ signal structure. Because type-$L$ agents are correct about the process generating the Outside signal and because the Outside signal does not impact the likelihood of political faith
preservation for type-R agents, the region of misaggregation of information where party R wins regardless of the state of the world is identical to the region presented in figure 6(a). In sum, whether we consider the asymmetry to be in exposure to Out-media or distrust in the quality of Out-media, the results are identical.

4.3 Non-common priors

In addition to holding political affiliations, it may be that agents hold more favorable priors towards their party, and we can very simply extend the baseline model to consider the implication of such non-common priors. Suppose that a type-R agent holds a prior $P_R[\omega = R] := w_R$, which is higher than that for a type-L agent ($P_L[\omega = R] := w_L$).

Figure 6(b) considers the case in which $w_R = 0.6$ and $w_L = 0.4$ and shows that the region of information aggregation failure, where party R wins regardless of the state of the world, is much larger with non-common priors than with common and symmetric priors ($P[\omega = R] := w = 0.5$) (the calculations are presented in Appendix B.3).

Here, if the perceived precision of the Outside signal ($t' = \tau t + \frac{(1-\tau)}{2}$) for a type-R agent is less than 0.6, then she can preserve her political faith regardless of the realization of the Outside signal by simply choosing a non-informative Inside signal structure. This is akin to a citizen consuming news commentary that has a political bias and the outlet does not claim to be providing journalistic facts. For higher levels of perceived precision of the Outside signal, the type-R agent would choose a GT or GB signal structure as before. Holding favorable priors allows type-R individuals to preserve their political faith more often.

Similar to type-R citizens, type-L citizens are also able to preserve their political faith more often when they hold priors favorable to their party. However, because they face a more informative Outside signal, the advantage their priors give them is limited. If the precision of the Outside signal is low enough for type-L agents to preserve their political faith always, then type-R agents will also be able to do the same. Furthermore, in the presence of non-common priors, for some parameter values, type-R agents always preserve their political faith while choosing a non-informative Inside signal structure, while type-L agents choose a GB signal structure that allows them to preserve their political faith with a probability that doesn’t equal one. Naturally, non-common priors would expand the parameter space within which we observe information misaggregation.

The assumption of agents sharing common priors reduces the region of information misaggregation, which is a key result of the model. In that context, we have tied our hands by assuming that agents share common priors.
In the next section, we consider the implications of a propagandized (biased) Outside signal for the agents’ choice of In-media as well as its effect on electoral outcomes.

5 Propaganda

A telltale sign of a decaying democracy is the state’s use of the mainstream media for propaganda purposes. We define propaganda as a bias in the Outside signal structure that stems from asymmetry in the realization of news that is favorable or unfavorable for political parties depending on the state of the world. We are specifically interested in the impact of propaganda on the agents’ choice of Inside-media as well as its effect on electoral outcomes. In particular, the model allows us to explore whether an agent’s ability to choose Inside-media can counteract propaganda, and furthermore, whether that ability can lead to information aggregation failure if the agent incorrectly believes that the mainstream media is biased.

Suppose that the Outside signal is biased in favor of party $L$ such that the signal $l$ is realized more often. Specifically

$$
P[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot 1,
\quad P[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot 0
$$

where $t \in [0.5, 1]$ is the precision of the Outside signal, and $\tau \in [0, 1]$ is the extent of unbiasedness in state-influenced media. The higher is $\tau$, the less-biased is the signal. This bias is commonly known by all agents and is the true process that generates the Outside signal.\footnote{The extent of unbiasedness, $\tau$, considered here is analogous to the probability with which a signal is credible, $\chi$, in Lipnowski et al. (2019). A similar notion of partial commitment is analyzed theoretically and experimentally in Fréchette et al. (2019).}

Such a signal structure for the Outside or mainstream media can exist when the state exerts control over mainstream media outlets. Suppose, for instance, that a strongman leader can force the mainstream media to run positive stories very frequently, but cannot prevent the occasional negative story from being run. In such an environment, he may also be able to censor media outlets to prevent agents from receiving an Inside signal. In the next two subsections, we therefore consider the implications of propaganda with and without censorship.
5.1 With censorship

Suppose that neither type of agent has access to any information other than her prior and the realized Outside signal. This scenario may be interpreted either as the party in power shutting down all media other than the propagandized state-controlled media or as a sparse media environment such as existed prior to the internet. We define this as a benchmark with censorship, which will help to highlight the role played by the ability to choose Inside media.

It is straightforward to show that because the agents share common and symmetric priors, and because the Outside signal is informative, the realization of the Outside signal determines whether the agent preserves her political faith. The condition for party $L$ to win in state $L$ is simply that the Outside signal $l$ be realized more often than the signal $r$, which, in fact, always holds (the details of this claim and the ones that follow are presented in appendix B.4). If the state of the world is $R$, then the Outside signal $l$ is realized more often if $\tau t < 0.5$. If this condition holds, then there is information misaggregation, and party $L$ wins regardless of the realization of the state of the world. This region is illustrated in figure 7(a). In the next subsection, we allow agents to design their chosen signal structure.

Figure 7: Propaganda with and without censorship

If the ruler of an illiberal democracy can influence the mainstream media and also prevent citizens from independently accessing information, then she can ensure electoral victory regardless of the state of the world for a large subset of the parameter space. Censorship along with propaganda is, therefore, a powerful combination of tools in an illiberal democracy.
5.2 Without censorship

Agents can now curate their own Inside signal structure. Being rational, they find an Outside signal that is favorable to the propagandizing side (party L) to be less informative than an Outside signal unfavorable to it. Recall that a Good signal from the Inside signal structure is designed to just counteract an unfavorable Outside signal. For type-R agents, the unfavorable Outside signal, \( l \), is relatively easy to counteract because it is less effective. For type-L agents, the unfavorable Outside signal is more difficult to counteract. As can be seen in figure 7(b), there is a large subset of parameters within which type-R agents optimally choose a GT signal structure and type-L agents choose a GB signal structure. The region of information misaggregation lies within this subset of parameter values.

In the region of information misaggregation, party R wins regardless of the state of the world. The intuition behind this result is that if the state of the world is \( R \), then party R must win because all type-R agents and some type-L agents vote for party R. If the state of the world is \( L \), then the informativeness of the Outside signal, \( r \), implies that an insufficient proportion of type-L agents preserve their political faith and vote for party L. For propaganda to backfire, it must be that the Out-media is sufficiently biased and that the precision of the Outside signal is strong enough to push type-L agents to choose a GB signal structure for their In-media. However, if the precision of the Out-media signal is high, then party L will win in state \( L \), and no misaggregation of information is present.

Table 2: Results with and without censorship

<table>
<thead>
<tr>
<th>Parameters: ( t = 0.6, \tau = 0.3 )</th>
<th>With censorship</th>
<th>Without censorship</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Ex-Ante} \omega = R ) ( \omega = L )</td>
<td>( \text{Ex-Ante} \omega = R ) ( \omega = L )</td>
<td></td>
</tr>
<tr>
<td>L Win Margin</td>
<td>+70%</td>
<td>+64%</td>
</tr>
<tr>
<td>L Win Prob</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 2 presents the case of \( t = 0.6 \) and \( \tau = 0.3 \) (see appendix B.4 and B.5 for the calculations). These parameter values correspond to a situation where the Outside signal is fairly precise, and the party-L–influenced media is known to be particularly biased. It can be seen that when there is censorship, party L can always win, and with high margins (reminiscent of the electoral results seen in some Eastern European “democracies” controlled by strongmen). On the other hand, without censorship, party L loses in both states of the world. Therefore, for a substantial subset of parameters, there is a reversal of electoral outcomes when censorship is disallowed. Suppose that party L cannot perfectly target propaganda, and that there is a likelihood of mistakes such that there is a positive probability for all levels of bias and precision. Then, party L must also institute
censorship in order to benefit from propaganda. Otherwise, with a positive probability, propaganda backfires.  

**Proposition 2** In the environment specified in section 5.2, the correct candidate wins except in a region with low to intermediate precision and a highly biased Outside signal. In the region of information misaggregation, type-R agents choose a GT signal structure, and type-L agents choose a GB signal structure.

The calculations and the proof can be found in appendix B.5.

Proposition 2 shows that without censorship, propaganda is not simply weak, it, in fact, backfires. Propaganda, by its very nature, implies that news favorable to the propagandizing party is discounted by Bayesian agents and does not affect their posteriors to a substantial degree. News unfavorable to the propagandizing party is particularly informative because it is so rare. If individuals can independently access information, then type-R agents need very little of it to counteract the propaganda in favor of party L. On the other hand, type-L agents need a much stronger signal to counteract unfavorable news from the biased media. For all parameters, party R wins in the state of the world it is meant to (state R), and for a substantial parameter space, it also wins in state L.

6 Misperceived Propaganda

We now consider two types of misperception of propaganda. In subsection 6.1, the ruler uses propaganda but agents, or a subset of them, nonetheless believe that the Out-media provides an unbiased signal. This might describe a situation in which the ruler of an illiberal democracy manages to coerce mainstream media into reporting propagandized news while also convincing the citizens that it is unbiased. Although the propagandizing side may have some advantage in this scenario, the fact that agents are not prevented from receiving In-media signals could counteract the propaganda. In subsection 6.2, we suppose that, although the Outside signal is, in fact, unbiased, agents of type R believe that it is biased in favor of party L. As noted earlier, this is an alternative interpretation of the asymmetric trust in mainstream media which appears to exist, for example, in the U.S. (see figure 2). The incorrect beliefs held by type R agents influence their choice of In-media and their belief updating, which may have important electoral consequences.

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19 Li et al. [2022] find that a truthful alternative media can counteract propaganda, which is a result that is similar in spirit.
6.1 Propaganda with citizens who are oblivious to it

We now consider an even less desirable situation in which the mainstream media is biased in favor of party \( L \) but citizens are convinced that it is unbiased. We first consider the case in which both types of agents are oblivious to the bias in the Outside signal. Later on, we consider cases in which only type-\( R \) or only type-\( L \) agents are oblivious to the bias. In each case, at least some citizens choose their In-media signal structure under incorrect beliefs. The mainstream media, which is biased toward party \( L \), generates a signal favorable to party \( L \) more often than one favorable to party \( R \).

The true process that generates the Out-media signal is biased in favor of party \( L \) and takes the following form:

\[
\begin{align*}
\mathbb{P}[s = l | \omega = L] &= \tau \cdot t + (1 - \tau) \cdot 1, \\
\mathbb{P}[s = r | \omega = R] &= \tau \cdot t + (1 - \tau) \cdot 0
\end{align*}
\]

where \( t \in [0.5, 1] \) is the precision of the Outside signal and \( \tau \in [0, 1] \) is the true level of unbiasedness of the Outside signal. The first case we consider is where both types of agents incorrectly believe that \( \tau = 1 \).

Figure 8: Signal choices and results in propaganda with oblivious citizens

![Signal choices and results with all citizens oblivious](https://example.com/signal_choices_all_oblivious.png)

![Results with only type-R citizens oblivious](https://example.com/results_only_type_R_oblivious.png)

(a) Signal choices and results with all citizens oblivious  
(b) Results with only type-\( R \) citizens oblivious

As can be seen in figure 8(a), both types of agents choose a \( GB \) signal structure for their In-media in the region of information aggregation failure (detailed calculations are provided in appendix B.6).

Although the agents are Bayesian, they update incorrectly and choose their In-media signal structure sub-optimally as a result of their incorrect beliefs. Type-\( R \) agents update excessively upon receiving an unfavorable Outside signal, and cushion themselves
against the Outside signal by choosing an overly informative Inside signal structure. They thus preserve their political faith less often than they could have. Similarly, type-L citizens update too little upon receiving unfavorable outside information — which works to their benefit. As we show in proposition 3 for low values of Out-media unbiasedness ($\tau$) and moderate to high values of Out-media precision ($t$), party $L$ enjoys such a large advantage that it can win regardless of the realized state of the world.

**Proposition 3** In the environment specified in subsection 6.1, the correct candidate wins, except in a region with an intermediate-precision and highly biased Outside signal. In the region of information misaggregation, both types of agents choose a GB signal structure.

The proof and detailed calculations are to be found in appendix B.6. The existence of such a region of information aggregation failure can explain why propagandizing outlets go to great lengths to portray themselves as accurate and balanced.

**Censorship:** We consider the implications of censorship in an environment where the Out-media is biased but citizens wrongly believe it is not. Party $L$ wins regardless of the state if the Out-media signal favorable to party $L$ is generated more often in either state of the world. As in subsection 5.1 this condition holds if $\tau t < 0.5$. The electoral results are, therefore, also identical and are illustrated figure 7(a). The only difference relative to subsection 5.1 lies in the intensity of beliefs that citizens hold about the parties for which they vote.

**$L$ agents are oblivious:** So far in this section, we have considered the case in which both types of players are oblivious to the bias in the Outside signal. Suppose, now, that only type-$L$ agents believe that the Outside signal is unbiased, while type-$R$ citizens know that the Outside signal is biased. Agents of type $L$ update less than they should when faced with an unfavorable Outside signal, and their Inside signal structure choice is the same as in figure 8(a). Agents of type $R$ choose their Inside signal structure as shown in figure 7(b). The correct side always wins in this specification, which for party $L$ is an improvement over the backfiring of propaganda we saw in subsection 5.2.

**$R$ agents are oblivious:** Now suppose that only agents of type $R$ are unaware of the bias in the Outside signal. Type-$L$ agents know that the Outside signal structure is biased in favor of Party $L$. Type-$L$ agents choose their Inside signal structure correctly, shown in figure 7(b). Type-$R$ agents hold incorrect beliefs and choose their Inside signal structure such that it is the same as in figure 8(a).
As shown in figure 8(b), the region where party $L$ wins regardless of the state of the world expands if only type-$R$ agents — rather than both types — are oblivious to the bias in the Outside signal. This demonstrates that for propaganda to benefit the ruler of an illiberal democracy, the opposing side must be unaware of it. Alternatively, the ruler can use censorship to win regardless of the state of the world.

### 6.2 Perceived propaganda

What happens if no actual propaganda is present yet some individuals believe there is? This obviously reflects distrust in the unbiasedness of the Outside media. Therefore, we now suppose that type-$R$ agents believe the Outside media to be biased when actually it is not.

Suppose the Outside signal structure is unbiased and has precision of $t \in [0.5, 1]$. Type-$R$ agents hold an incorrect belief and perceive the media to be biased when it is not. Thus, they believe that the Outside signal structure follows:

$$\Pr[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad \Pr[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot 0$$

where $\tau \in [0, 1]$ is type-$R$ agents’ belief in the extent of the Outside signal’s unbiasedness. Type-$L$ agents correctly believe that the process that generates the Outside signal is unbiased, which is equivalent to them believing that $\tau = 1$.

**Figure 9: Signal choices with perceived propaganda**

![Signal choices with perceived propaganda](image)

Figure 9 shows the Inside signal structure choices for each type of agent as well as the region of information aggregation failure. In this region, type-$R$ agents perceive the
media to be highly biased, and the Outside signal is at least moderately precise. The intuition behind this result is similar to that provided in subsection 5.2 — namely, the perception of bias in the Outside signal structure makes an unfavorable Outside signal less informative for type-R agents. Therefore, asymmetry in the perception of bias by the mainstream gives party $R$ an advantage that allows it to win regardless of the state of the world for a subset of parameters (the calculations are provided in appendix B.7).

**Proposition 4** In the environment specified in subsection 6.2, the correct candidate wins except in a region where the Outside signal has intermediate precision and low perceived unbiasedness. In the region of information misaggregation, type-R agents choose a GT signal structure and type-L agents choose a GB signal structure.

We find that the perception of propaganda is a strong political tool that can allow the *incorrect* party to win. One implication is that party $R$ has an incentive to convince type-R voters that the mainstream media is biased in favor of party $L$. As noted in the introduction, President Trump railed against the mainstream media, accusing it of being biased. Our model suggests that if type-R voters believe that the mainstream media is biased in favor of party $L$, then that will influence the In-media choices of type-R voters. This gives party $R$ a major electoral advantage, and for a substantial subset of parameters party $R$ can win regardless of the state of the world.

Here, we considered an interpretation of media distrust in which some agents incorrectly believe that the media is *biased* and found that there exists a substantial region of information aggregation. In subsection 4.2, we interpreted distrust in the mainstream media as an incorrect belief held by type $R$ citizens regarding the Outside signal’s *precision* and found a comparable result. Considering the similarities in the regions of information misaggregation in figures 9 and 3(a), it is apparent that the phenomenon of an electoral benefit accruing to the side with greater distrust of mainstream media is robust to our interpretation of mistrust.

### 7 Conclusion

Many have been wondering how electoral outcomes in liberal democracies, such as the election of Trump, Bolsonaro or the Brexit vote, could have happened and whether the current rich media environment played a key role in such elections. By contrast in non-liberal democracies, such as Russia or Hungary among many others, where the government fully controls the official media message, the executive also engages in a costly and extensive crack down (explicitly or implicitly) on non-government media, suggesting that
such media may counteract effectively government propaganda, namely that propaganda of the state alone is not effective enough in persuading its citizens.

To understand the role of an ever richer media environment in democracies, we have explored how instrumental media choice driven by political identity can affect aggregate electoral results. In the core setup, bias is assumed to exist in the preferences that drive media choice but is not cognitive. Specifically, all individuals update rationally using all the information they receive and vote according to that information. We later added cognitive biases to see how that would change the results.

For liberal democracies, such as the United States, our benchmark assumes symmetry between the two sides, where the asymmetry exists only in the exposure to Outside media or in the bias of Outside media. In those contexts, we showed electoral outcomes can be affected and highlighted situations in which information aggregation fails. When that occurs, one candidate wins in all states of the world — even when she is the worse candidate, or in other words, even when average/expected rational beliefs following any media signal — chosen or not — are unfavorable to her. A key determinant of this failure of information aggregation is that one side has low exposure to mainstream media or low trust in it, while the other side has a moderate level of exposure/trust. In this region, the introduction of a rich media environment can provide a winning margin advantage to the side less exposed to mainstream media to the point that it can swing elections in all states of the world. This misaggregation region (characterized by low and moderate exposure on Right and Left sides of the political spectrum, respectively) seems particularly salient in democracies such as the U.S., perhaps explaining why some parties have an incentive to encourage distrust in the mainstream media. If we add in cognitive biases such as biased priors, this misaggregation region becomes even larger.

In the case of illiberal democracies, we found that for government propaganda to work, it is crucial that individuals be unaware of it. In other words, individuals also have a cognitive bias. If they are aware of the bias, then the presence of a free media undermines government propaganda spectacularly. Indeed, propaganda may backfire entirely, allowing the non-propagandizing side to win regardless of the state of the world. This may explain why it is not sufficient for authoritarian governments to control the official media message and to promote government propaganda if citizens are aware of it. For such governments to maintain power, they must limit the free media and dissenting news outlets so that possibly inconvenient truths are not disseminated.
References


A Solution for the general model

Suppose that an agent believes (whether correctly or incorrectly) that
\[ P[s = l | \omega = L] = k, \quad P[s = r | \omega = R] = m \]

The agent’s expected utility still follows equation [1] and the signal structure must satisfy the Martingale constraint.

The agent’s interim posteriors on observing the Outside signal and before observing the signal from the chosen signal structure are such that:
\[ P[\omega = L | s = l] = \frac{k}{1 + k - m}, \quad P[\omega = L | s = r] = \frac{1 - k}{1 + m - k} \]

The G, B, and T signals are described section 3. A type-R agent is able to preserve her political faith as long has her posterior upon observing both signals is such that \( P[\omega = R | s, S] \geq 0.5 \). Therefore, the G, B, and T signals must be such that:
\[
\begin{align*}
P[\omega = R | S = G] &= \frac{k}{1 + k - m} \\
P[\omega = R | S = B] &= \frac{1 - k}{1 + m - k} \\
P[\omega = R | S = T] &= 0
\end{align*}
\]

Claim 1 It is sufficient to consider three signals used in two possible signal structures when solving the agent’s problem described above.

Proof. The proof follows directly from Kolotilin (2018) and Kamenica & Gentzkow (2011). We use the linear optimization technique of comparing marginal utility to price ratios (MU-Price ratios) for the different signals. In figure 3, the MU-Price ratio of a signal is represented by the slope of the line from the origin to the point on the expected utility curve that corresponds to the interim posterior generated by that signal. In that sense, the rest of the proof are simply confirms the shape of the concave closure of the expected utility curve.

Lemma 2 Any signal \( M_i \) that generates a posterior \( P[\omega = R | M_i] \in \left( \frac{k}{1 + k - m}, 1 \right] \) is sub-optimal when compared to a signal \( M_G \) where \( M_G \) is such that \( P[\omega = R | M_G] = \frac{k}{1 + k - m} \).

Proof. Regardless of whether the agent observes \( M_i \) or \( M_G \), her expected utility will be the same. This is because the agent is able to preserve her political faith regardless of the realization of the Outside signal.
However, it is more costly (according to the Martingale constraint) to generate the signal $M_i$. Therefore, the MU-Price ratio of generating $M_i$ is less than that for generating $M_G$. This implies that any signal structure where $P[M_i] > 0$ will have a lower ex-ante expected utility than a signal structure that assigns $P[M_i] = 0$ and adds $P[\omega = R|M_i]P[M_i]$ to the probability that $M_G$ is generated. Therefore, no $M_i$ such that $P[\omega = R|M_i] \in (\frac{k}{1+k-m}, 1]$ will be chosen by the agent. Equivalently, $M_i$ is sub-optimal when compared to $M_G$.

**Lemma 3** Any signal $M_i$ that generates a posterior $P[\omega = R|M_i] \in (0, \frac{1-k}{1+m-k})$ is sub-optimal when compared to a signal $M_B$ where $M_B$ is such that $P[\omega = R|M_B] = \frac{1-k}{1+m-k}$.

**Proof.** For $M_i$, the ex-ante expected utility of the agent is given by:

$$P\left[P[\omega = R|M_i, s] \geq 0.5\right] = mP[\omega = R|M_i] + (1-k)(1-P[\omega = R|M_i])$$

$$= 1-k + P[\omega = R|M_i](m + k - 1)$$

This implies that the MU-Price ratio is:

$$(m + k - 1) + \frac{(1-k)}{P[\omega = R|M_i]}$$

For $M_B$, the ex-ante expected utility is:

$$P\left[P[\omega = R|M_B, s] \geq 0.5\right] = m\left(\frac{1-k}{1+m-k}\right) + (1-k)(1-\left(\frac{1-k}{1+m-k}\right)) = 2m\left(\frac{1-k}{1+m-k}\right)$$

This means that the MU-Price ratio is:

$$2m = (m + k - 1) + \frac{(1-k)}{\left(\frac{1-k}{1+m-k}\right)}$$

Since $P[\omega = R|M_i] > \frac{1-k}{1+m-k}$, the MU-Price ratio for generating a signal structure posterior $M_i$ is lower than for $M_B$. Therefore, no $M_i \in (\frac{1-k}{1+m-k}, \frac{k}{1+k-m})$ will be chosen by the agent. Equivalently, $M_i$ would be sub-optimal when compared to $M_B$.

**Lemma 4** Any signal $M_i$ that generates a posterior $P[\omega = R|M_i] \in (0, \frac{1-k}{1+m-k})$ is sub-optimal when compared to a signal $M_T$ where $M_T$ is such that $P[\omega = R|M_T] = 0$.

**Proof.** For $M_i$, the ex-ante expected utility is zero. This is because regardless of the realization of the Outside signal, the agent is never able to preserve her political faith, and
the same is true for \( M_T \). However, \( M_i > 0 \), which implies that the cost for generating \( M_i \) is higher than that for generating \( M_T \). Therefore, no \( M_i \in (0, \frac{1-k}{1+m-k}) \) will be chosen by the agent. Equivalently, \( M_i \) is sub-optimal when compared to \( M_T \). ■

The agent requires only three signals to solve her problem. In fact, any signal that generates a posterior different from them would be sub-optimal. \( M_G \), which generates a posterior \( P[\omega = R|M_G] = \frac{k}{1+k-m} \), is abbreviated to \( G \). \( M_B \), which generates a posterior \( P[\omega = R|M_B] = \frac{k}{1+m-k} \), is abbreviated to \( B \). Finally, \( M_T \), which generates a posterior \( P[\omega = R|M_T] = 0 \), is abbreviated to \( T \).

A signal structure is a combination of signal realizations, and the three possible signals are \( G, B, \) and \( T \). Therefore, the possible signal structures are \( GT, GB, \) and \( GBT \).20

We argue that while \( GBT \) is feasible according to the budget constraint, and might even be an optimal choice for some parameters, it can be ignored, since whenever it is optimal, a simpler signal structure (\( GB \) or \( GT \)) is as well. In other words, this signal structure never offers strictly greater expected utility (than the max of \( GB \) and \( GT \)), and is therefore not required to solve the agent’s problem. Either the MU-Price ratio of \( G \) is higher than that of \( B \) in which case \( GT \) should be implemented, rather than \( GBT \); or the MU-Price ratio of \( B \) is higher than that of \( G \) in which case \( GB \) should be implemented, rather than \( GBT \); or, the MU-Price ratios of \( G \) and \( B \) are equal, in which case either \( GB \) or \( GT \) provides the agent with the same expected utility as \( GBT \), and therefore, \( GBT \) can be ignored.

Therefore, \( GT \) and \( GB \) alone are sufficient to solve the agent’s problem. ■

### A.1 The type-\( R \) agent’s problem

Recall that:

\[
P[s = l|\omega = L] = k, \quad P[s = r|\omega = R] = m
\]

This is a linear optimization problem, and therefore, the agent chooses to employ the signals with the highest MU-Price ratio.

For signal \( G \), the MU is 1. This is because regardless of the Outside signal, the agent is able to preserve her political faith. For signal \( B \), the MU is equal to the likelihood that the Outside signal is favorable (\( r \), for a type-\( R \) agent) given that \( B \) is realized, i.e. \( 2m(\frac{1-k}{1+m-k}) \).

Finally, for \( T \), the agent is never able to preserve her political faith, and therefore the MU

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20There are a number of signal structures that are ruled out because they violate the Martingale constraint, specifically, \( G, B, T, \) and \( BT \). While we assume that the agents share a common symmetric prior belief that \( P[\omega = R] = 0.5 \), this result is robust to values of \( P[\omega = R] \) such that \( \frac{1-k}{1+m-k} < P[\omega = R] < \frac{k}{1+k-m} \).
is 0.

The price of each of these signals is determined according to the coefficient corresponding to it in the Martingale constraint, i.e. \((\frac{k}{1+k-m}) \cdot P_G + (\frac{1-k}{1+m-k}) \cdot P_B + 0 \cdot P_T = 0.5\). This price is simply the intermediate posterior generated by the signal.

The MU-Price ratio is \(\frac{1}{k} \) for signal \(G\), \(\frac{2m}{(1+m-k)}\) for signal \(B\), and undefined for signal \(T\).

The signal structure \(GT\) is optimal when MU-Price ratio for signal \(G\) is at least as large as that for signal \(B\), which simplifies to \(1 + k - m - 2km \geq 0\). If \(1 + k - m - 2km \leq 0\), then signal structure \(GB\) is optimal. This is equivalent to saying that the concave closure of the expected utility curve shown in figure 4 has a kink if and only if \(1 + k - m - 2km < 0\). If it does, then a \(GB\) signal provides the agent with a higher expected utility than a \(GT\) signal.

We can now calculate the probability of realization of different signals, the utility achieved, and the likelihood of voting for the preferred party under the signal structures \(GT\) and \(GB\).

The probabilities of realizing the different signals will help us calculate expected utilities as well as the outcome of the election.

**Signal structure \(GT\):** Unconditional on the state, the likelihood that the signal \(G\) is realized is \(P_G = \frac{1+k-m}{2k}\), which is also the agent’s ex-ante expected utility.

Conditional on the state being \(\omega = R\), the signal \(G\) is always realized and therefore the agent’s expected utility is \(E[U_{R|GT}\cap\omega = R] = P[G|\omega = R] = 1\).

Conditional on the state being \(\omega = L\), the likelihood that signal \(G\) is realized is \(\frac{1-m}{k}\). Whenever signal \(G\) is realized, the agent is able to preserve her political faith. Therefore, this also equals the agent’s expected utility \((E[U_{R|G\cap\omega = L}]\)

**Signal structure \(GB\):** This signal structure is somewhat more complicated, and therefore, we use the following three equations.

\[
P[G|GB] + P[B|GB] = 1 \tag{2}
\]
\[
P[G|\omega = L] = \frac{1-m}{k} \tag{3}
\]
\[
P[G|\omega = R] = \frac{1}{k} \tag{4}
\]
\[
P[B|\omega = L] = \frac{m}{1-k} \tag{4}
\]

42
Given that the signal structure is $GB$, 

$$1 = P[G] + P[B]$$

$$= \left( P[G|\omega = R] \cdot \frac{1}{2} + P[G|\omega = L] \cdot \frac{1}{2} \right) + \left( P[B|\omega = R] \cdot \frac{1}{2} + P[B|\omega = L] \cdot \frac{1}{2} \right)$$

$$= \frac{1}{2} \left( (P[G|\omega = R]) \left( 1 + \frac{P[G|\omega = L]}{P[G|\omega = R]} \right) + (P[B|\omega = R]) \left( 1 + \frac{P[B|\omega = L]}{P[B|\omega = R]} \right) \right)$$

which simplifies to:

$$1 = P[G|\omega = R] \frac{1 + k - m}{2k} + (1 - P[G|\omega = R]) \frac{1 + m - k}{2(1 - k)}$$

Therefore, conditional on the state,

$$P[G|GB \cap \omega = R] = k, \quad P[B|GB \cap \omega = R] = 1 - k$$

$$P[G|GB \cap \omega = L] = 1 - m, \quad P[B|GB \cap \omega = L] = m$$

and unconditional on the state:

$$P[G|GB] = \frac{1 + k - m}{2}, \quad P[B|GB] = \frac{1 + m - k}{2}$$

To calculate the likelihood of political faith preservation, and therefore, expected utility, it is helpful to recall that

$$P[s = l|\omega = L] = k, \quad P[s = r|\omega = R] = m$$

The agent’s expected utility conditional on the state being $\omega = R$ is:

$$E[U_R|GB \cap \omega = R] = P[G|GB \cap \omega = R] + P[r|GB \cap \omega = R] \cdot P[B|GB \cap \omega = R]$$

$$= k + m (1 - k)$$

while the agent’s expected utility conditional on the state being $\omega = L$ is:

$$E[U_R|GB \cap \omega = L] = P[G|GB \cap \omega = L] + P[r|GB \cap \omega = L] \cdot P[B|GB \cap \omega = L]$$

$$= (1 - m) + (1 - k)m$$

The unconditional expected utility is simply a weighted average of the conditional expected utilities. If the agent’s beliefs about the signal structure of the Outside signal are
correct, then:

\[
E[U_R|GB] = \frac{1}{2} \cdot E[U_R|GB \cap \omega = R] + \frac{1}{2} \cdot E[U_R|GB \cap \omega = L] \\
= \frac{1 + k + m - 2km}{2}
\]

A.2 The type-\(L\) agent’s problem

Recall that:

\[
P[s = l|\omega = L] = k, \quad P[s = r|\omega = R] = m
\]

The intuition behind the calculations is similar to the type-\(R\) agent’s problem. Furthermore, all one needs to do to arrive at these calculations is to use the calculations from the previous subsection, and replace \(w\) with \(1 - w\) and \(k\) with \(m\).

The MU for signal \(G\) is 1. For signal \(B\), the MU is \(2k\left(\frac{1-m}{1+k-m}\right)\). Finally, for signal \(T\), the MU is 0. The price of each of these signals is determined according to the to coefficient corresponding to it in the Martingale constraint \(\left(\frac{m}{1+m-k}\right) \cdot P_G + \left(\frac{1-m}{1+k-m}\right) \cdot P_B + 0 \cdot P_T = 0.5\).

This price is simply the intermediate posterior generated by the signal. The MU-Price ratio is \(\frac{1}{1+m-k}\) for signal \(G\), \(\frac{2k(\frac{1-m}{1+k-m})}{(\frac{1+m-k}{1+k-m})}\) for signal \(B\), and undefined for signal \(T\).

The signal structure \(GT\) is optimal when \(1 + m - k - 2km \geq 0\). If \(1 + m - k - 2km \leq 0\), then signal structure \(GB\) is optimal.

We can now calculate the probability of realization of different signals, the utility achieved, and the likelihood of voting for the preferred party under the signal structures \(GT\) and \(GB\).

**Signal structure \(GT\):** Unconditional on the state, the likelihood that the signal \(G\) is realized is \(P[G] = \frac{1+m-k}{2m}\), which is also the agent’s ex-ante expected utility.

Conditional on the state being \(\omega = L\), the signal \(G\) is always realized and so the agent’s expected utility is \(E[U_L|GT \cap \omega = L] = P[G|\omega = L] = 1\).

Conditional on the state being \(\omega = R\), the likelihood that signal \(G\) is realized is \(\frac{1-k}{m}\). Whenever it is realized, the agent is able to preserve her political faith. Therefore, this also equals the agent’s expected utility \(E[U_L|G \cap \omega = R]\).
Signal structure $GB$: This signal structure is somewhat more complicated, and therefore, we use the following three equations.

\[
P[G|GB] + P[B|GB] = 1 \tag{5}
\]

\[
P[G|\omega = R] = \frac{1 - k}{m} \tag{6}
\]

\[
P[B|\omega = R] = \frac{k}{1 - m} \tag{7}
\]

Conditional on the state,

\[
P[G|GB \cap \omega = L] = m, \quad P[G|GB \cap \omega = R] = 1 - k
\]

and unconditional on the state:

\[
P[G|GB] = \frac{1 + m - k}{2}, \quad P[B|GB] = \frac{1 + k - m}{2}
\]

To calculate the likelihood of political faith preservation, and therefore expected utility, it is helpful to recall that:

\[
P[s = l|\omega = L] = k, \quad P[s = r|\omega = R] = m
\]

The agent’s expected utility conditional on the state being $\omega = L$ is:

\[
E[U_L|GB \cap \omega = L] = P[G|GB \cap \omega = L] + P[l|B \cap \omega = L] \cdot P[B|GB \cap \omega = L]
\]

\[
= m + k(1 - m)
\]

The agent’s expected utility conditional on the state being $\omega = R$ is:

\[
E[U_L|GB \cap \omega = R] = P[G|GB \cap \omega = R] + P[l|B \cap \omega = R] \cdot P[B|GB \cap \omega = R]
\]

\[
= (1 - k) + (1 - m)k
\]

The unconditional expected utility is simply a weighted average of the conditional expected utilities. If the agent’s beliefs about the signal structure of the Outside signal are
correct, then:

\[
E[U_{L|GB}] = \frac{1}{2} \cdot E[U_{L|GB\cap\omega=R}] + \frac{1}{2} \cdot E[U_{L|GB\cap\omega=L}]
= \frac{1+k+m-2km}{2}
\]

### A.3 Robustness to noise

Figure 10: Expected utility as a function of interim posteriors

(a) Lower precision Out-media with noise

(b) Higher precision Out-media with noise

We now show that the key results of the model remain unchanged if we incorporate a small amount of noise in the agents’ threshold for political faith preservation. This is very similar to and follows directly from Extension A in *Kamenica & Gentzkow (2011)* in which the authors find that the stochasticity in the receiver’s action means that the sender’s expected payoff function is smooth, and that the key results are unchanged.

Suppose that the belief threshold that type-R agents have for the preservation of their political faith is stochastic, with mean \(P[\omega = R|S_i,s_i] = 0.5\). The noise in the threshold implies that the agent’s preservation of her political faith is now stochastic, and varies continuously with the agent’s interim posterior. The smoothness of the agent’s expected utility function in figure 10 is due to this stochasticity.

If there is a small amount of mean-zero, normally distributed noise in this threshold, then for a less precise Outside signal structure, as can be seen in figure 10, a \(G^+T\) signal is optimal. The \(G^+\) signal generates an interim posterior that is more favorable to party R than a G signal and in the case of normally distributed noise, it is optimal because
it allows the agent to preserve her political faith for a large proportion of the possible realizations of the noise.

Similarly, for high-precision Outside signals, a $G^+B^+$ signal structure is optimal for type-$R$ agents. For minute levels of noise, the optimal signal structures remain almost identical and would result in almost the same signal structures being chosen.

Since the key results in propositions 1, 2, 3, and 4 were not knife-edge, they are robust to stochasticity in the belief threshold, as long as that the variance of the noise is low.

**A.4 Robustness to gain from being correct**

We now show for the general model that the key results are robust to including a small gain from being correct in the agents’ utility function. In other words, in addition to a gain from political faith preservation, agents also gain utility from being correct about the state of the world. The utility function of a type-$R$ agent is, therefore, modified to become:

$$U_R = \begin{cases} 
(1 - \gamma) + \gamma, & \text{if } \mathbb{P}[\omega = R|S,s] \geq 0.5 \text{ and } \omega = R \\
(1 - \gamma), & \text{if } \mathbb{P}[\omega = R|S,s] \geq 0.5 \text{ and } \omega = L \\
\gamma, & \text{if } \mathbb{P}[\omega = R|S,s] < 0.5 \text{ and } \omega = L \\
0, & \text{if } \mathbb{P}[\omega = R|S,s] < 0.5 \text{ and } \omega = R 
\end{cases} \tag{8}$$

In this case, $\gamma \in (0,1)$ captures the extent to which the agent gains utility from being correct as opposed to a signal that allows her to preserve her political faith.\textsuperscript{21}

We now show two key results for a type-$R$ agent. First, for a low value of $\gamma$, $GT$ and $GB$ signal structures are sufficient to solve the agent’s problem. Second, as $\gamma$ increases, the threshold at which the agent switches from a $GT$ signal structure to a $GB$ signal structure changes continuously. These two results together imply that the key results of the model are robust to small values of $\gamma$. For an agent of type $L$, the same results hold analogously.

Based on the above utility function, there are four interim posteriors (generated after the agent observes her Inside signal, and before she observes the Outside signal) that are key to solving the agent’s problem. Three are derived from the $G$, $B$, and $T$ signals, while

\textsuperscript{21}$\gamma = 0$ is the benchmark model. $\gamma = 1$ corresponds to a case in which the agent only wants to know the correct state. In that case, the agent will choose a fully revealing echo chamber signal structure.
the fourth is an *Excellent* or *E* signal. The signals must be such that:

\[ P[\omega = R|S = G] = \frac{k}{1 + k - m} \]
\[ P[\omega = R|S = B] = \frac{1 - k}{1 + m - k} \]
\[ P[\omega = R|S = T] = 0 \]
\[ P[\omega = R|S = E] = 1 \]

The set of signal structures that satisfy the Martingale property is:

\[ \{ET, GT, EB, GB, EBT, EGT, GBT, EGBT\} \]

We disregard the signal structures *EBT, EGT, GBT*, and *EGBT* because whenever one of them is optimal, a simpler signal structure will also be.

In the next step, we compare the ex-ante expected utilities of each of these signal structures to show that for low values of \( \gamma \), *GT* and *GB* are sufficient to solve the agent’s problem.

**Signal structure ET:**

\[ E[U_R|_{ET \cap \omega = R}] = 1, \quad E[U_R|_{ET \cap \omega = L}] = \gamma \]
\[ E[U_R|_{ET}] = \frac{1 + \gamma}{2} \]

**Signal structure EB:**

\[ E[U_R|_{EB \cap \omega = R}] = 1 - \frac{(1 - k)(1 - m)}{m}, \quad E[U_R|_{EB \cap \omega = L}] = 1 - \gamma - m + 2\gamma m \]
\[ E[U_R|_{EB}] = 1 - \frac{1}{2} \left[ \frac{(1 - k)(1 - m)}{m} + \gamma + m - 2\gamma m \right] \]

**Signal structure GT:**

\[ E[U_R|_{GT \cap \omega = R}] = 1, \quad E[U_R|_{GT \cap \omega = L}] = \gamma + \frac{1 - m}{k} - 2\gamma \frac{1 - m}{k} \]
\[ E[U_R|_{GT}] = \frac{1 + \gamma}{2} + \left( \frac{1 - m}{k} \right) \left( \frac{1}{2} - \gamma \right) \]
Signal structure $GB$:

$$E[U_R|_{GB\cap \omega = R}] = k + m - km, \quad E[U_R|_{GB\cap \omega = L}] = \gamma + (1 - 2\gamma)(1 - km)$$

$$E[U_R|_{GB}] = \frac{1 - \gamma + k + m}{2} - (1 - \gamma)km$$

Claim 5 Suppose that $\gamma \leq 0.5$. The signal structures $GT$ and $GB$ are then sufficient to solve the agent’s problem.

Proof. We now show that the signal structure $GT$ provides the agent with at least as much expected utility as $ET$ or $EB$ as long as $\gamma \leq 0.5$.

$$E[U_R|_{GT}] - E[U_R|_{ET}] \geq 0$$ simplifies to $\left(\frac{1-m}{k}\right)\left(\frac{1}{2} - \gamma\right) \geq 0$, and holds if $\gamma \leq 0.5$.

Similarly, $E[U_R|_{GT}] - E[U_R|_{EG}] \geq 0$ simplifies to $(1-m)(1-k)\left(\frac{1-2\gamma}{k} + \frac{1}{k}\right) \geq 0$, and holds if $\gamma \leq 0.5$. ■

Because $E[U_R|_{GT}]$ and $E[U_R|_{GB}]$ are continuous functions of $\gamma$, the agent’s choice of Inside signal structure depends on a threshold that varies continuously with $\gamma$. Therefore, for low values of $\gamma$, the region of information aggregation failure doesn’t disappear completely, although it may shrink.

A.5 Robustness to gain from holding more favorable posteriors

We now show that the if the agents also gain utility from holding posteriors that are more favorable to their preferred party, then for all levels of that gain, the results are identical.

The utility function of a type-$R$ agent is now modified to become:

$$U_R = \begin{cases} 
(1 - \lambda) + \lambda P[\omega = R|S, s] & \text{, if } P[\omega = R|S, s] \geq 0.5 \\
\lambda P[\omega = R|S, s] & \text{, if } P[\omega = R|S, s] < 0.5 
\end{cases}$$

As in the earlier setup, the agent gains a utility from preserving her political faith. She also gains some utility from holding favorable posteriors. Here, $\lambda \in [0, 1)$ captures the weight that the agent places on holding more favorable posteriors, while $1 - \lambda$ is the agent’s utility from preserving her political faith.\footnote{\(\lambda = 0\) is the benchmark model. \(\lambda = 1\) corresponds to a case in which the agent’s utility is linear in how favorable her posterior belief is towards her party.}

Based on the above utility function, there are three interim posteriors (generated after the agent observes her Inside signal, and before she observes the Outside signal) which
are key to solving the agent’s problem, and which are the same as in the benchmark model:

\[
\begin{align*}
P[\omega = R | S = G] &= \frac{k}{1+k-m} \\
P[\omega = R | S = B] &= \frac{1-k}{1+m-k} \\
P[\omega = R | S = T] &= 0
\end{align*}
\]

As in the benchmark model, only two signal structures are required to solve the agent’s problem, i.e. GT and GB. Furthermore, the trade-off between the two signals remains unchanged. Specifically, the agent chooses GT if the MU-Price ratio of the G signal is at least as large as that of the B signal. This simplifies to:

\[
\begin{align*}
\frac{\text{MU}_G}{P_G} &= \frac{\lambda \frac{k}{1+k-m} + (1-\lambda)}{1 + m - k} \\
&\geq \frac{\text{MU}_B}{P_B} \\
\lambda + \frac{(1-\lambda)(1+k-m)}{k} &\geq \lambda + (1-\lambda)(2m) \\
1 + k - m - 2km &\geq 0
\end{align*}
\]

This condition is identical to the one in the benchmark model. Therefore, the agent’s choice of Inside signal structure remains unchanged in this modified model. This also implies that the results of the modified model are identical to those of the benchmark for all values of \(\lambda \in [0,1)\).

**B Applications**

**B.1 Asymmetric exposure to an unbiased Outside signal**

A type-\(L\) agent receives an Outside signal such that:

\[
P[s = l | \omega = L] = t \in [0.5, 1], \quad P[s = r | \omega = R] = t \in [0.5, 1]
\]

A type-\(R\) agent receives a less precise Outside signal:

\[
P[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1], \quad P[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1]
\]
where $\tau \in [0, 1]$.

B.1.1 The type-$R$ agent’s problem

We use the results developed in appendix A.1 and simply plug in $k = m = \frac{1-\tau}{2} + \tau t$.

Signal structure $GT$: This signal structure is chosen if $\tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq \frac{1}{\sqrt{2}} \approx 0.71$.

The agent’s expected utilities are the same as her likelihood of political faith preservation and are equal to:

$$E[U_R|_{GT \cap \omega = R}] = 1, \quad E[U_R|_{GT \cap \omega = L}] = \frac{1 + \tau - 2\tau t}{1 - \tau + 2\tau t}$$

Unconditioned on the realization of the state, the ex-ante expected utility is

$$E[U_R|_{GT}] = \frac{1}{1 - \tau + 2\tau t}$$

Signal structure $GB$: This signal structure is chosen if $\tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \geq \frac{1}{\sqrt{2}} \approx 0.71$.

The agent’s expected utilities are the same as the likelihood of political faith preservation for the agent, and they equal:

$$E[U_R|_{GB \cap \omega = R}] = \frac{3}{4} + \tau t - \frac{\tau^2}{2} - \left(\frac{1}{4} + t^2 - t\right), \quad E[U_R|_{GB \cap \omega = L}] = 1 - \left(\frac{1 - \tau + 2\tau t}{2}\right)^2$$

Unconditioned on the realization of the state, the ex-ante expected utility is

$$E[U_R|_{GB}] = \frac{3}{4} - \tau^2 \left(t^2 + \frac{1}{4} - t\right)$$

B.1.2 The type-$L$ agent’s problem

Signal structure $GT$: This signal structure is chosen if $t \leq \frac{1}{\sqrt{2}} \approx 0.71$.

$$E[U_L|_{GT \cap \omega = R}] = \frac{1 - t}{t}, \quad E[U_L|_{GT \cap \omega = L}] = 1$$

Signal structure $GB$: This signal structure is chosen if $t \geq \frac{1}{\sqrt{2}} \approx 0.71$.

$$E[U_L|_{GB \cap \omega = R}] = 1 - t^2, \quad E[U_L|_{GB \cap \omega = L}] = 2t - t^2$$
B.1.3 Proof of proposition

**Proof.** Suppose that $\tau = 0.04$ and $t = 0.75$. A type-$R$ agent chooses a GT signal structure because $\tau \cdot t + (1 - \tau) \cdot \frac{1}{2} = 0.51 < \frac{1}{\sqrt{2}}$, while type-$L$ agent chooses a GB signal structure because $t = 0.75 > \frac{1}{\sqrt{2}}$. These conditions hold in the neighborhood of the parameter values $\tau = 0.04$ and $t = 0.75$. Since the likelihood of political faith preservation is continuous in these parameter values, and since the margin of victory for party $R$ in state $L$ at these parameter values is bounded away from zero, the result of information aggregation failure holds for a non-trivial subset of values.

If both types of agents choose a GT signal structure, and if the state of the world is $R$, then all type-$R$ agents vote for party $R$, and some type-$L$ agents also vote for party $R$. If the state of the world is $L$, then all type-$L$ agents vote for party $L$ and some type-$R$ agents also vote for party $L$. Clearly, the correct party wins in either state.

Furthermore, there is no parameter space in which type-$R$ agents choose a GB signal structure while type-$L$ agents choose a GT signal structure.

If both types of agents choose a GB signal structure and if the state of the world is $R$, then party $R$ wins if:

\[
\frac{3}{4} + \tau t - \frac{\tau}{2} - \tau^2 \left(\frac{1}{4} + t^2 - t\right) > 1 - t^2
\]

which simplifies to:

\[
\tau \left(t - \frac{1}{2}\right) \left(1 - \tau \left(t - \frac{1}{2}\right)\right) + \left(t^2 - \frac{1}{4}\right) > 0
\]

which always holds.

Similarly, if the state of the world is $L$, then party $L$ wins if:

\[
1 - \left(\frac{1 - \tau + 2\tau t}{2}\right)^2 < 2t - t^2
\]

which simplifies to:

\[
\left(\frac{1 - \tau + 2\tau t}{2}\right)^2 - (1 - t)^2 > 0
\]

This always holds because $1 - t \in [0, 0.5]$ while $\left(\frac{1 - \tau + 2\tau t}{2}\right) \in [0.5, 1]$. ■
B.2 Distrust in the mainstream media

Suppose that type-\(R\) agents believe (incorrectly) that the media is less precise than it actually is. Specifically, type-\(L\) agents correctly believe that the process generating the Outside signal is such that:

\[
P[s = l | \omega = L] = t \in [0.5, 1], \quad P[s = r | \omega = R] = t \in [0.5, 1]
\]

In contrast, type-\(R\) agents incorrectly believe that the process that generates the Outside signal is more noisy, such that:

\[
P[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1], \quad P[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \in [0.5, 1]
\]

where \(\tau \in [0, 1]\)

This is similar to an asymmetry in exposure to mainstream media. As such, the signal choices and expected utilities are identical to those calculated in subsection B.1.\(^{23}\)

For type-\(L\) agents, the probability of political faith preservation is identical to the expected utilities calculated in subsection B.1.

For type-\(R\) agents, if the chosen signal is of type \(GT\), then the probability of political faith preservation is identical to the expected utility calculated in subsection B.1. If the chosen signal is of type \(GB\), then a type-\(R\) agent’s probability of political faith preservation is calculated conditioned on the state.

If the state is \(\omega = R\), then:

\[
P[PFP_{R|GB \cap \omega = R}] = P[G|GB \cap \omega = R] + P[r|B \cap \omega = R] \cdot P[B|GB \cap \omega = R]
\]

\[
= \left( \frac{1 - \tau}{2} + \tau t \right) + t \cdot \left( \frac{1 + \tau}{2} - \tau t \right)
\]

\[
= \frac{1}{2} \left( 1 - \tau + t + 3\tau t - 2\tau t^2 \right)
\]

If the state is \(\omega = L\), then:

\[
P[PFP_{R|GB \cap \omega = L}] = P[G|GB \cap \omega = L] + P[r|B \cap \omega = L] \cdot P[B|GB \cap \omega = L]
\]

\[
= \left( \frac{1 + \tau}{2} - \tau t \right) + (1 - t) \cdot \left( \frac{1 - \tau}{2} + \tau t \right)
\]

\[
= \frac{1}{2} \left( 2 - t + \tau t - 2\tau t^2 \right)
\]

\(^{23}\)For type-\(R\) agents, the expectations are based on incorrect beliefs. We will, therefore, separately calculate the probability of political faith preservation.
B.3 Non-common priors

Suppose that type-R and type-L agents have different priors. Specifically,

\[ P_R[\omega = R] := w_R \quad \text{and} \quad P_L[\omega = R] := w_L \]

where \( w_L < 0.5 < w_R \).

As in subsection 4.1, a type-L agent receives an Outside signal, such that:

\[ P[s = l | \omega = L] = t \in [0.5, 1], \quad P[s = r | \omega = R] = t \in [0.5, 1] \]

A type-R agent receives a less precise Outside signal, such that:

\[ P[s = l | \omega = L] = \tau \cdot t + \left(1 - \tau\right) \cdot \frac{1}{2} \in [0.5, 1], \quad P[s = r | \omega = R] = \tau \cdot t + \left(1 - \tau\right) \cdot \frac{1}{2} \in [0.5, 1] \]

where \( \tau \in [0, 1] \)

B.3.1 The type-R agent’s problem

We use the results developed in appendix A.1 and allow for \( w_R > 0.5 \) while plugging in \( k = m = \frac{1 - \tau}{2} + \tau t \).

**Signal structure N:** This signal structure is chosen if \( \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq w_R \)

The agent is able to preserve her political faith regardless of the realization of the Outside signal.

\[ E[U_R | N \cap \omega = R] = 1, \quad E[U_R | N \cap \omega = L] = 1 \]

**Signal structure GT:** This signal structure is chosen if \( w_R < \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \leq \frac{1}{\sqrt{2}} \simeq 0.71 \).

The agent’s expected utilities are the same as her likelihood of political faith preservation, and are equal to:

\[ E[U_R | GT \cap \omega = R] = 1, \quad E[U_R | GT \cap \omega = L] = \frac{w_R(1 + \tau - 2\tau t)}{(1 - w_R)(1 - \tau + 2\tau t)} \]

**Signal structure GB:** This signal structure is chosen if \( \tau \cdot t + (1 - \tau) \cdot \frac{1}{2} \geq max\{\frac{1}{\sqrt{2}}, w_R\} \).
The agent’s expected utilities are the same as her likelihood of political faith preservation, and are equal to:

\[
E[U_R|GB \cap \omega = R] = \frac{(2\tau t + 1 - \tau)(2\tau t - 1 - \tau)}{4w_R \tau(2t - 1)} + \frac{(2\tau t + 1 - \tau)(2\tau t + 1 - \tau - 2w_R)(1 + \tau - 2\tau)}{8w_R \tau(2t - 1)}
\]

and

\[
E[U_R|GB \cap \omega = L] = \frac{(1 + \tau - 2\tau t)(2\tau t - 1 - \tau)}{4(1 - w_R) \tau(2t - 1)} + \frac{(2\tau t + 1 - \tau)(2\tau t + 1 - \tau - 2w_R)(1 + \tau - 2\tau)}{8(1 - w_R) \tau(2t - 1)}
\]

**B.3.2 The type-\(L\) agent’s problem**

**Signal structure \(N\):** This signal structure is chosen if \(t \leq 1 - w_L\).

The agent is able to preserve her political faith regardless of the realization of the Outside signal.

\[
E[U_L|N \cap \omega = R] = 1, \quad E[U_L|N \cap \omega = L] = 1
\]

**Signal structure \(GT\):** This signal structure is chosen if \(1 - w_L < t \leq \frac{1}{\sqrt{2}} \approx 0.71\).

\[
E[U_L|GT \cap \omega = R] = \frac{(1 - w_L)(1 - t)}{w_L t}, \quad E[U_L|GT \cap \omega = L] = 1
\]

**Signal structure \(GB\):** This signal structure is chosen if \(t \geq \max\{\frac{1}{\sqrt{2}}, 1 - w_L\}\).

\[
E[U_L|GB \cap \omega = R] = \frac{(1 - t)(t - w)}{w(2t - 1)} + \frac{t(1 - t)(t + w - 1)}{w(2t - 1)}
\]

and

\[
E[U_L|GB \cap \omega = L] = \frac{t(2t - 1 - t^2 + (1 - w)t)}{(1 - w)(2t - 1)}
\]

**B.4 Propagandized Outside signal with censorship**

Here that the Outside signal is biased towards party \(L\), which is common knowledge. The structure of the Outside signal is:

\[
\mathbb{P}[s = l|\omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad \mathbb{P}[s = r|\omega = R] = \tau \cdot t + (1 - \tau) \cdot 0
\]

where \(t > 0.5\).
We can use the results from appendix A.1 and A.2 by simply plugging in \( k = 1 + \tau t - \tau \) and \( m = \tau t \).

Because we are considering censorship, neither type of agent has access to any information other than her prior and the realized Outside signal. The agents share common and symmetric priors, and their posteriors on observing the Outside signals are:

\[
Pr[\omega = L | s = l] = \frac{1 + \tau t - \tau}{2 - \tau}
\]

Since \( t > 0.5 \), \( Pr[\omega = L | s = l] > 0.5 \) and

\[
Pr[\omega = L | s = r] = 1 - t
\]

Since \( t > 0.5 \), \( Pr[\omega = L | s = r] < 0.5 \)

For party \( L \) to win in state \( R \), we require that \( Pr(s = l | \omega = R) > Pr(s = r | \omega = R) \) or

\( 1 - \tau t > \tau t \). That is, \( \tau t < 0.5 \).

For party \( L \) to win in state \( L \), we require that \( Pr(s = l | \omega = L) > Pr(s = r | \omega = L) \) or

\( 1 + \tau t - \tau > \tau - \tau t \). That is, \( \tau (1 - t) < 0.5 \). If \( \tau \in [0, 1] \), and \( t > 0.5 \), or if \( \tau \in (0, 1] \) and \( t \geq 0.5 \) then this always holds.

Therefore, party \( L \) can win in both states of the world if \( \tau t < 0.5 \).

B.5 Propagandized Outside signal without censorship

Here, that the Outside signal is biased towards party \( L \), and this bias is common knowledge. The structure of the Outside signal is:

\[
P[s = l | \omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad P[s = r | \omega = R] = \tau \cdot t + (1 - \tau) \cdot 0
\]

where \( t > 0.5 \).

We can use the results from appendix A.1 and A.2 by simply plugging in \( k = 1 + \tau t - \tau \) and \( m = \tau t \).

B.5.1 The type-\( R \) agent’s problem

GT signal structure: Chosen if \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \geq 0 \).

The agent’s expected utilities are:

\[
E[U_R |_{GT \cap \omega = R}] = 1, \quad E[U_R |_{GT \cap \omega = L}] = \frac{1 - \tau t}{1 + \tau t - \tau}
\]
**GB signal structure:** Chosen if $2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \leq 0$.

The agent’s expected utilities are:

$$E[U_R | GB \cap \omega = R] = 1 - \tau + \tau t + \tau^2 t - \tau^2 t^2, \quad E[U_R | GB \cap \omega = L] = 1 - \tau t + \tau^2 t - \tau^2 t^2$$

**B.5.2 The type-L agent’s problem**

**GT signal structure:** Chosen if $\tau(1 - 2t - 2\tau t^2 + 2\tau t) \geq 0$ The agent’s expected utilities are:

$$E[U_L | GT \cap \omega = R] = \frac{1 - t}{t}, \quad E[U_L | GT \cap \omega = L] = 1$$

**GB signal structure:** Chosen if $\tau(1 - 2t - 2\tau t^2 + 2\tau t) \leq 0$ The agent’s expected utilities are:

$$E[U_L | GB \cap \omega = R] = 1 - \tau t + \tau^2 t - \tau^2 t^2, \quad E[U_L | GB \cap \omega = L] = 1 - \tau t + \tau^2 t - \tau^2 t^2$$

**B.5.3 Proof of proposition**

**Proof.** Suppose that $\tau = 0.3$ and $t = 0.6$. Type-R agents choose a GT signal structure because $2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t = 1.3832 > 0$, while a type-L agents choose a GB signal structure because $\tau(1 - 2t - 2\tau t^2 + 2\tau t) = -0.0168 < 0$. These conditions hold in the neighborhood of the parameter values $\tau = 0.3$ and $t = 0.6$. Since the likelihood of political faith preservation is continuous in these parameter values, and since the margin of victory for party R in state L at these parameter values is bounded away from zero, the result of information aggregation failure holds for a non-trivial subset of values.

If both types of agents choose a GT signal structure, and if the state of the world is R, then all type-R agents vote for party R, and some type-L agents also vote for party R. If the state of the world is L, then all type-L agents vote for party L and some type-R agents also vote for party L. Clearly, the correct party wins in either state.

There is no parameter space where type-R agents choose a GB signal structure while type-L agents choose a GT signals structure.

If both types of agents choose a GB signal structure, and if the state of the world is R, then party R wins if:

$$1 - \tau + \tau t + \tau^2 t - \tau^2 t^2 > 1 - \tau t + \tau^2 t - \tau^2 t^2$$

which simplifies to:
\[ \tau(2t - 1) > 0 \]

which always holds.

Similarly, if the state of the world is \( L \), then party \( L \) wins if:

\[
1 - \tau + \tau t + \tau^2 t - \tau^2 t^2 > 1 - \tau t + \tau^2 t - \tau^2 t^2
\]

which simplifies to:

\[
\tau(2t - 1) > 0
\]

which always holds. \( \blacksquare \)

**B.6 Propaganda with oblivious citizens**

In this case, that the true process of the Outside signal is:

\[
P[s = l|\omega = L] = \tau \cdot t \vee (1 - \tau) \cdot 1, \quad P[s = r|\omega = R] = \tau \cdot t \vee (1 - \tau) \cdot 0
\]

where \( \tau \in [0, 1] \) and \( t \in [0.5, 1] \). Agents don’t know that the true process, and believe that the process generating the Outside signal is unbiased, such that:

\[
P[s = l|\omega = L] = t, \quad P[s = r|\omega = R] = t
\]

**B.6.1 The type-\( R \) agent’s problem**

*GT signal structure:* This signal structure is chosen if \( t \leq \frac{1}{\sqrt{2}} \)

The agent’s expected utilities are:

\[
E[U_R|GT \cap \omega = R] = 1, \quad E[U_R|GT \cap \omega = L] = \frac{1 - t}{t}
\]

If the agent chooses a \( GT \) signal structure, then the likelihood of political faith preservation does not depend on the realization of the Outside signal. Therefore, the likelihood of political faith preservation is identical to the expected utility.
**GB signal structure:** This signal structure is chosen if \( t \geq \frac{1}{\sqrt{2}} \). The agent’s expected utilities (under incorrect beliefs) are:

\[
E[U_R|GB \cap \omega = R] = 2t - t^2, \quad E[U_R|GB \cap \omega = L] = 1 - t^2
\]

Here, the likelihood of political faith preservation differs from the agent’s expected utility.

\[
P[PFP_R|GB \cap \omega = R] = t(1 + \tau - \tau t), \quad P[PFP_R|GB \cap \omega = L] = (1 + \tau t)(1 - t)
\]

### B.6.2 The type-L agent’s problem

**GT signal structure:** This signal structure is chosen if \( t \leq \frac{1}{\sqrt{2}} \).

The agent’s expected utilities are:

\[
E[U_L|GT \cap \omega = R] = \frac{1 - t}{t}, \quad E[U_L|GT \cap \omega = L] = 1
\]

If the agent chooses a GT type signal structure, then the likelihood of political faith preservation does not depend on the realization of the Outside signal. Therefore, the likelihood of political faith preservation is identical to the expected utility.

**GB signal structure:** This signal structure is chosen if \( t \geq \frac{1}{\sqrt{2}} \).

The agent’s expected utilities (under incorrect beliefs) are:

\[
E[U_L|GB \cap \omega = R] = 1 - t^2, \quad E[U_L|GB \cap \omega = L] = 2t - t^2
\]

In this case, the likelihood of political faith preservation differs from the agent’s expected utility, such that:

\[
P[PFP_L|GB \cap \omega = R] = 1 - \tau t^2, \quad P[PFP_L|GB \cap \omega = L] = 1 - \tau(1 - t)^2
\]

### B.6.3 Proof of proposition 3

**Proof.** Suppose that \( \tau = 0.1 \) and \( t = 0.75 \). Both types of agents choose a GB signal structure for their In-media because \( t > \frac{1}{\sqrt{2}} \). These conditions hold in the neighborhood of the parameter values \( \tau = 0.1 \) and \( t = 0.75 \). Since the likelihood of political faith preservation is continuous in these parameter values, and since the margin of victory for party L in state R at these parameter values is bounded away from zero, the result of information aggregation failure holds for a non-trivial subset of values.
If both types of agents choose a GT signal structure, and if the state of the world is \( R \), then all type-\( R \) agents vote for party \( R \), and some type-\( L \) agents also vote for party \( R \). If the state of the world is \( L \), then all type-\( L \) agents vote for party \( L \) and some type-\( R \) agents also vote for party \( L \). Clearly, the correct party wins in either state.

There is no parameter space in which the two types of agents choose different signal structures for their respective In-media consumption.

If both types of agents choose a GB signal structure, and if the state of the world is \( L \), then party \( L \) wins if:

\[
1 - \tau(1 - t)^2 > (1 + \tau t)(1 - t)
\]

This simplifies to:

\[
t + \tau t - \tau > 0
\]

which always holds.

Furthermore, if the state of the world is \( R \), then party \( L \) wins if:

\[
1 - \tau t^2 > t(1 + \tau - \tau t)
\]

which simplifies to

\[
t < \frac{1}{1 + \tau}
\]

Therefore, party \( L \) wins regardless of the state if \( t \in \left( \frac{1}{\sqrt{2}}, \frac{1}{1 + \tau} \right) \). There are no parameter values for which party \( R \) can win in state \( L \).

B.7 Perceived propaganda

Suppose that the true process of the Outside signal is:

\[
P[s = l|\omega = L] = t \in [0.5, 1], \quad P[s = r|\omega = R] = t \in [0.5, 1]
\]

Type-\( L \) agents know the true process, while type-\( R \) agents believe that the process of the Outside signal is biased in the following way:

\[
P[s = l|\omega = L] = \tau \cdot t + (1 - \tau) \cdot 1, \quad P[s = r|\omega = R] = \tau \cdot t + (1 - \tau) \cdot 0
\]

where \( \tau \in [0, 1] \).
B.7.1 The type-R agent’s problem

**GT signal structure**: This signal structure is chosen if \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \geq 0 \).

The agent’s expected utilities are:

\[
E[U_R|_{GT} \land \omega=R] = 1, \quad E[U_R|_{GT} \land \omega=L] = \frac{1 - \tau}{1 + \tau t - \tau}
\]

If the agent chooses a GT signal structure, then the likelihood of political faith preservation does not depend on the realization of the Outside signal. Therefore, the likelihood of political faith preservation is identical to the expected utility.

**GB signal structure**: This signal structure is chosen if \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \leq 0 \).

The agent’s expected utilities (under incorrect beliefs) are:

\[
E[U_R|_{GB} \land \omega=R] = 1 - \tau + \tau t + \tau^2 t - \tau^2 t^2, \quad E[U_R|_{GB} \land \omega=L] = 1 - \tau t + \tau^2 t - \tau^2 t^2
\]

In this case, the likelihood of political faith preservation differs from the agent’s expected utility, such that:

\[
\mathbb{P}[PFP_R|_{GB} \land \omega=R] = 1 - \tau + 2\tau t - \tau t^2, \quad \mathbb{P}[PFP_R|_{GB} \land \omega=L] = 1 - \tau t^2
\]

B.7.2 The type-L agent’s problem

**GT signal structure**: This signal structure is chosen if \( t \leq \frac{1}{\sqrt{2}} \).

The agent’s expected utilities are:

\[
E[U_L|_{GT} \land \omega=R] = \frac{1 - t}{t}, \quad E[U_L|_{GT} \land \omega=L] = 1
\]

**GB signal structure**: This signal structure is chosen if \( t \geq \frac{1}{\sqrt{2}} \).

The agent’s expected utilities are:

\[
E[U_L|_{GB} \land \omega=R] = 1 - t^2, \quad E[U_L|_{GB} \land \omega=L] = 2t - t^2
\]

B.7.3 Proof of proposition 4

**Proof.** Suppose that \( \tau = 0.1 \) and \( t = 0.75 \). A type-R agent chooses a GT signal structure because \( 2 - \tau - 2\tau t - 2\tau^2 t^2 + 2\tau^2 t \approx 1.75 > 0 \). A type-L agent chooses a GB signal structure because \( t = 0.75 > \frac{1}{\sqrt{2}} \). These conditions hold in the neighborhood of the
parameter values $\tau = 0.1$ and $t = 0.75$. Since the likelihood of political faith preservation is continuous in these parameter values, and since the margin of victory for party $R$ in state $L$ at these parameter values is bounded away from zero, the result of information aggregation failure holds for a non-trivial subset of values.

If both types of agents choose a $GT$ signal structure, and if the state of the world is $R$, then all type-$R$ agents vote for party $R$, and some type-$L$ agents also vote for party $R$. If the state of the world is $L$, then all type-$L$ agents vote for party $L$ and some type-$R$ agents also vote for party $L$. Clearly, the correct party wins in either state.

There is no parameter space where type-$R$ agents choose a $GB$ signal structure while type-$L$ agents choose a $GT$ signals structure.

If both types of agents choose a $GB$ signal structure, and if the state of the world is $R$, then party $R$ wins if:

$$1 - \tau + 2\tau t - \tau t^2 > 1 - t^2$$

This simplifies to:

$$\tau (2t - 1) + t^2 (1 - \tau) > 0$$

which always holds.

Similarly, if the state of the world is $L$, then party $L$ wins if:

$$2t - t^2 > 1 - \tau t^2$$

which simplifies to:

$$(2t - 1) - t^2 (1 - \tau) > 0$$

which holds for values of $t \in \left[\frac{1}{\sqrt{2}}, 1\right]$ and $\tau \in \left[\frac{2}{3}, 1\right]$. The region where both types of agents choose a $GB$ signal structure is a subset of the region where $t \in \left[\frac{1}{\sqrt{2}}, 1\right]$ and $\tau \in \left[\frac{2}{3}, 1\right]$. Therefore, if both types of agents choose a $GB$ signal structure, then the correct party wins. ■