Brexit: Compromise Under Threat

Helios Herrera* Antonin Macé † Matías Núñez ‡

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Abstract

We study how a walk-away threat ending negotiations affects welfare and gridlock in the ratification of deals/treaties. In every period, an agreement needs to be ratified by two opposing parties. Agreement failure provokes either an extension and a (freshly renegotiated) amended agreement to be ratified or a "hard outcome" (worse than any possible deal) and an end to the negotiation. A walk-away threat can be announced strategically by either party (or by third parties). We show that such threats do improve the scope for agreement, but also entail costs. In the symmetric case, only highly credible threats are beneficial: when an agreement is unlikely to begin with, threats with low credibility only reduce welfare by increasing the equilibrium chance of a hard outcome. In the general case, the advantaged party typically benefits from walk-away threats even for low credibility threats, as these shift the expected agreement in his favor. The disadvantaged should threaten to walk-away only if highly credible.

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*University of Warwick & CEPR.

†CNRS, Paris School of Economics & École Normale Supérieure.

‡CNRS, CREST & École Polytechnique.

"Brinkmanship...the threat that leaves something to chance" (Thomas Schelling)

1 Introduction

Since the Brexit referendum in June 2016 the everlasting negotiations for a withdrawal agreement with the EU (finally ratified in the UK in Jan. 2020) marked a low point in British democracy. For three times the UK Parliament rejected a negotiated agreement in early 2019, which then had to be renegotiated in Brussels. Every time, PM Theresa May, despite threatening not to do so, requested last minute deadline extensions (which were granted by the EU) to avoid a no-deal Brexit that would significantly hurt the UK economy. Some experts came to believe that Brexit could be delayed forever, and viewed Parliament as not the solution to Brexit but the problem itself. Indeed, the possibility of a re-vote in the future on a new deal fostered the unwillingness to compromise of UK parties and factions. To try to solve what became known as the "kicking the can down the road problem" *threats* of no extension to the ratification deadlines (precipitating no-deal Hard Brexit outcome) were made on several occasions by several EU countries, notably France and Ireland.[1] On the UK side, in the summer 2019, PM Boris Johnson also vowed for the UK to leave the EU by Oct 31 2019 "do or die" pledging not to extend the deadline.[2] Only an early election with a Tory landslide broke the impasse and allowed the withdrawal agreement to be ratified in Jan. 2020 by the UK. But the saga just moved to a second, not less dramatic, stage, namely agreeing on a UK-EU trade deal, once again under a tight "do or die" deadline. The pound had one of its worse week of 2019 after on Dec. 16 Boris Johnson legislated a deadline for the UK's Brexit transition period, pledging to outlaw any extension to the UK's post-Brexit transition period beyond the end of 2020. The tightness of this agreement time-window is unprecedented for

[1]For instance, from Ireland (see later) and from France (e.g. see The Guardian Oct 28 2019: "Macron against Brexit extension as Merkel keeps option open").

[2]Boris Johnson tried also to suspend the UK parliament (provoking a constitutional crisis), thus presenting to the UK MPs a Hard Brexit on Oct. 31 as the only alternative to the current agreement. See The Economist Aug. 29 2019: "Taking Back Control".

any yet-to-be negotiated trade deal.[3] Unlike the domestic episode of October 2019, this time the UK's walk-away threat is targeted to the counterparts in Brussels.[4]

Political negotiations or treaty ratifications are usually subject to deadlines only extendable under certain conditions and/or approval by external parties. The threat of a hard outcome if the deadline is not met/extended surely has the common objective of breaking the impasse by forcing the negotiating parties to compromise more and reduce costly delays, but can also be imposed unilaterally and strategically by parties inside or outside the negotiation, as the cases above show. Crucially, these threats may work because they create the *risk* of an accident: rather than fully credible threats they generate probabilistic outcomes that hang over the negotiation like a Damocles's sword. In fact, these announcements must be credible in part at least as they have substantial effects financial markets. The extent of their credibility depends on the credibility of the threatening side as well as other random factors. As Schelling (1960) observes: "the key to these threats is that, though one may or may not carry them out if the threatened party fails to comply, the final decision is not altogether under the threatener's control....these risks could involve chance, accident, third-party influence, imperfection in the machinery of decision, or just processes that we do not entirely understand."

These brinkmanship episodes beg both normative and positive questions. Namely, when is imposing such a burden upon a negotiation welfare improving, which side would be willing to impose such walk-away threat, if any, and how credible should this threat be to be advantageous. Prima facie, there seems to be two possible benefits of walk-away threats, i.e., major costs to failure/delay of agreement. One is a common benefit: making both sides more willing to compromise thus reducing the cost of extended negotiations. The other is private: gaining a negotiating advantage possibly when the threat of a hard outcome hurts more one side than the other. On the flip

[3]Overall, Brexit no-deal scenarios have continued to affect sterling since the beginning. (see e.g. https://www.ft.com/content/5452f2f8-4672-11ea-paraee2-9ddbdc86190d). As things stand today Britain will exit the EU single market and customs union at the end of 2020 and trade according to WTO global rules only, unless the UK asks for a deadline extension. Once again, no extension would hurt much more the UK than its counterpart the EU.

[4]See for instance The Economist May 28th 2020 "Brexit: Deadlock looms at Brexit talks next week: the chances Britain will leave the EU without a trade deal are rising".

side, there are costs of imposing such threats if the hard outcome de facto materializes thereby hurting, possibly to a different extent, both parties.

To shed light on the above trade-offs, we present a model in which two parties must repeatedly decide to ratify or not, a proposed agreement presented to them. These proposed agreements are randomly drawn every period out of a bounded distribution, which reflects the unavoidable underlying uncertainty on how proposals are generated from the point of view of the body that decides its final approval.[5] If a proposal is rejected then, with some probability $h \geq 0$, this causes the game to end with a *hard outcome*, which is ruinous, possibly to different extents, to either side. If an extension is granted then a new period starts in which new proposal is drawn to be voted on, and so forth. The core model we analyze is a pure private values, constant sum game or pie-sharing situation (in the absence of a hard outcome). In our setup, h is the probability that a rejected deal prompts an end to negotiations thus a hard outcome, while $(1 - h)$ represents the chance that the negotiating process continues through one additional period/proposal. Crucially, h can be manipulated by parties by means of announcements that threaten not extending the negotiation for additional periods if the current deal is rejected. The extent of this manipulation depends on how credible these parties' threats may be. Our premise is that these "do or die" announcements that parties can make are, in general, only partly credible. Namely, while it is politically costly to renege on a "do or die" announcement, there are also clear incentives not to carry through with the threat if, ex-post, a deal is not reached and an extension is needed. Taking as exogenous credibility of a "do or die" announcement, our goal is to understand when and why parties would make such threats and how this affects several outcomes: welfare of the two sides, the per period chance of a deal/delay, the overall equilibrium chance of a hard outcome. The two compromising sides may differ crucially in their disutility from the hard outcome.

[5] Our focus is the final approval/ratification of an agreement previously negotiated by a committee/delegation. This negotiation may entail bargaining between several factions, inside and/or outside the economy, as well as unforeseen economic and unanticipated institutional constraints becoming binding. In the case of Brexit, no political actor knew exactly what future proposed agreement is in store next if the current withdrawal agreement is turned down, or what EU-UK trade deal will end up being ratified, if any. The ratification of a negotiated agreement may fail in general. For instance, the Trans-Pacific Parnership (TPP), signed by all twelve negotiating countries in 2016, never came into effect because most countries did not ratify it at home.

We find that the unique stationary equilibrium is characterized by an *agreement set*, which represents the scope for agreement, namely the deals acceptable by both parties. In general, a larger threat of no-extension h (weakly) enlarges the equilibrium scope for agreement making a deal more likely to be accepted in every period: a higher h always succeeds in improving the chances of agreement forcing parties to compromise more.[6] While this could have been anticipated, the effects on welfare are more subtle. Namely, a larger h is always (weakly) effective in enlarging the scope for agreement (agreement set), but at the same time the threat may materialize: the hard outcome may become more likely in equilibrium, which in turn reduces welfare.

We start by analyzing a symmetric version of the model in which both parties would be equally affected by a hard outcome. We show that regardless of the parameters and the size of threats, welfare depends only on one sufficient statistic: the equilibrium agreement probability or scope for agreement l, albeit in a convex, generically non-monotone way. This implies that, if we start from a default situation in which agreements are unlikely, then low credibility threats do increase the agreement scope but only make things worse for either parties: only threats that are highly credible can improve welfare. Thus, in this case if either side or third party has no ability/credibility to make h high enough, it should avoid threats all together. We show that stepping up pressure marginally is welfare improving only if the deal is more likely than not: if an agreement is *less likely than not* ($l < 1/2$) to begin with, then announcements/threats with low credibility are counterproductive, only large threats help, but if the agreement is more likely than not ($l > 1/2$) to begin with then *any* additional threat helps. This is somewhat surprising because additional threats are most effective in improving the scope for agreement if this is small to begin with. At the same time though, this is also when the equilibrium chance of a hard outcome increases the most.

Low credibility threats can be rationalized in the asymmetric model though, as we show. If one party is advantaged in the sense that it perceives a lower cost of the hard outcome then it may use even low credibility threats to shift the whole agreement set

[6] Evidently, for a high enough h all agreements are accepted which implements the first best in terms of total welfare: no delay and no hard outcome in equilibrium. This amounts to an ex-ante commitment of both parties to accept immediately the first deal put on the table.

more to his advantage. We show that as h increases the agreement set moves gradually through three qualitatively different regimes: two-sided compromise, one-sided compromise and full agreement. Only in the two-sided compromise region advantaged party has incentives to increase (at least marginally) the walk-away threat, while this is never the case in the other two regions. Indeed, in the first region the threat shifts the expected agreement more to the side of the advantaged party. As for the disadvantaged party, a low credible threat is never a good idea: a threat makes sense only if, despite its disadvantage, it is credible enough to get close to provoking an immediate agreement.

Our model speaks also of the rationale behind threats made by third parties, which have no part in the negotiation but may have the power to extend the negotiation or end it and may threaten to do so (as e.g. the EU in the case of Brexit[7]) for their benefit, or, in a normative interpretation, for total welfare. Third parties in general may have different payoffs from the negotiating parties, both for expected agreements and for hard outcomes, so threats to make time run out on negotiations depend on these payoffs and may be counterproductive for the welfare of the two negotiating parties. In general, low credibility threats are enough to maximize the welfare of a third party with positive hard outcome payoffs.

In the following, after the literature review, we introduce the model, analyzing the symmetric case before the general case and some extensions. All proofs are relegated to the appendix.

2 Related Literature

This paper touches on several strands of literature, which we outline below.

Ratification. Conceptually, our work speaks to the interaction between an executive branch which negotiated an agreement (possibly with an outside party/country) and the legislative branch that needs to ratify it. For instance, Humphreys [2007] studies strategic ratification touching upon the seminal ideas of Putnam [1988] and

[7] Besides the case of France mentioned above, see also the case of Ireland: https://www.bbc.co.uk/news/world-europe-50101428

Schelling [1960]. However, we look at this interaction once a deal has been negotiated, not before, thus for instance at how an executive branch, who has the power to seek extensions to deadline and renegotiation, can put pressure on the legislative branch who has the power to ratify the current deal.

Collective search. Our modeling strategy borrows from the collective search and experimentation models, in which a group chooses every period between accepting the current negotiation outcome or wait for a new outcome next period[8]. For instance, Compte and Jehiel [2010] show that more stringent majority requirements select more efficient proposals but take more time to do so and Albrecht et al. [2010] find that committees are more permissive than a single decision maker facing an otherwise identical search problem.[9] Compte and Jehiel [2017] push further the same approach for large committees characterizing the optimal majority rule. Also, Strulovici [2010] and Messner and Polborn [2012] focus on committee decisions in which preferences are unknown and only learned over time, thus the option to delay happens in equilibrium albeit with different degrees of efficiency depending on the majority rule. Moldovanu and Rosar [2019] study voting in a Brexit-like model with one irreversible option and compare the effect of different voting rules. They show that voting by supermajority over two consecutive periods dominates voting by simple majority.

Stochastic bargaining. In our model offers/deals are exogenous, but there is a vast literature of legislative bargaining models with endogenous offers in which elements of stochasticity generate inefficient delays in agreements or gridlock in the presence of an endogenous status quo. Several papers analyze stochastically evolving preferences, see Dziuda and Loeper [2016] or Bowen et al. [2017]. Other works explore the case of delay with a stochastic total surplus, such as Eraslan and Merlo [2002], Merlo and Wilson [1998], Merlo and Wilson [1995].

Timing games. Lastly several authors have looked at the effect of hard deadlines

[8]This literature is somehow related to a classic literature on bargaining where players are allowed to search for outside options, see Wolinsky [1987] and Chikte and Deshmukh [1987] for classic treatments on the question. See Muthoo [1995] that analyzes the role of players being able to leave temporarily the negotiation and Manzini and Mariotti [2004] where bilateral bargaining where players can agree on a joint outside option is considered.

[9]In a related model with common values, Moldovanu and Shi [2013] study costly search for a committee and studies how acceptance thresholds and welfare depend on the degree of conflict within the committee.

in negotiations, which is not our focus in our stationary setup. Namely, while we study dynamic negotiation between two parties in the presence of a stationary stochastically extendable deadline, in most of the literature, the deadline is tight in the sense that no extension is possible. This generates incentives to reach agreements in the "eleventh hour", that is at or very close to the deadline (see Simsek and Yildiz [2016] for the role of optimism in these models). Such (non-stationary) timing games have been studied by Fuchs and Skrzypacz [2013] and others[10].

3 Model

Two agents are bargaining over a set of possible *deals* $X = [0, 1]$. Each agent is characterized by its bliss point $\theta \in \{0, 1\}$, and has a linear utility on X:

$$\forall x \in X, \qquad u_\theta(x) = 1 - |\theta - x|.$$

The final outcome of the bargaining may be a deal in X or the *hard outcome* d. The option d does not lie in the set X and yields a utility $u_\theta(d) = d_\theta \in (-\infty, 0)$ for each agent $\theta \in \{0, 1\}$. We denote by $D = d_0 + d_1$ the hard outcome's *total value*, and by $B = d_1 - d_0$ the hard outcome's *bias*. Without loss of generality, we assume that $B \geq 0$, and if $B > 0$, we refer to agent 1 as the *advantaged* agent and to agent 0 as the *disadvantaged* one.

The bargaining procedure takes place sequentially. At each period $t \in \mathbb{N}$, a proposed deal $x_t \in X$ is drawn from the uniform distribution on X, independently from the previous draws. In other words, agents do not control the agenda which is random. Then, agents simultaneously choose to accept or reject the proposal. If both accept it at period t, the final outcome is x_t. Otherwise, a Bernoulli variable H of parameter $h \in (0, 1)$ is drawn independently of previous draws. If $H = 1$, the procedure stops and the outcome is the hard outcome d, obtained at period t. If $H = 0$, an extension is granted and both players move to the next period $t + 1$. The parameter h represents the *threat*, that is, the probability that the hard outcome is

[10]See Cramton and Tracy [1992] for empirical evidence or Güth et al. [2001] for experimental evidence on this observation.

implemented at each period. Finally, utilities are discounted with a common discount factor $\beta \in (0,1)$.

The strategy of an agent consists in accepting or rejecting deals as they arrive, and hence it could depend on the history of play. We restrict our attention to stationary equilibria, for which agents' strategies are time-independent. For a given stationary strategy profile, we denote by $A \subseteq X$ its *agreement set*, i.e. the set of deals that are accepted if proposed, and by w_θ agent θ's *reservation value*, i.e. his expected utility when he rejects a deal. By stationarity, reservation values satisfy the following recursive equation:

$$w_\theta = \overbrace{hd_\theta}^{\text{hard outcome } d} + \overbrace{\beta(1-h)\mathbb{P}(x \in A)\mathbb{E}[u_\theta(x) \mid x \in A]}^{x \text{ lies in the agreement set}} + \overbrace{\beta(1-h)\mathbb{P}(x \notin A)w_\theta}^{x \text{ does not lie in the agreement set}} .$$

For a stationary strategy profile to be an equilibrium, each agent must accept a deal $x \in X$ if and only if its utility exceeds the agent's reservation value, i.e. $u_\theta(x) \geq w_\theta$. Thus, the condition for the profile to be an equilibrium is that its agreement set satisfies:

$$A = A_w = \{x \in X \mid u_\theta(x) \geq w_\theta,\ \forall \theta \in \{0,1\}\}.$$

The condition for a stationary strategy profile with reservation values $w = (w_0, w_1)$ to be an equilibrium can thus be summarized by:

$$\forall \theta \in \{0,1\}, \qquad w_\theta = \frac{hd_\theta + \beta(1-h)\mathbb{P}(x \in A_w)\mathbb{E}[u_\theta(x) \mid x \in A_w]}{1 - \beta(1-h)\mathbb{P}(x \notin A_w)}. \tag{1}$$

Building on equation (1), we now derive a first result showing that, under the assumptions concerning the bargaining procedure, a stationary equilibrium exists. For a stationary equilibrium w, we denote by c_w the *center* of the agreement set A_w and by l_w its *length*, so that we can write: $A_w = [c_w - l_w/2, c_w + l_w/2]$. In the sequel, we write A, c and l for simplicity.

Theorem 1 *A stationary equilibrium w exists. Moreover, at each such equilibrium,* *the agreement set A is a non-empty closed interval with center $c \geq 1/2$.*

The result's proof, as well as all proofs, are included in the appendix.

4 Symmetric Model

We now describe equilibrium outcomes in the symmetric model (when $B = 0$).

4.1 Equilibrium

Our first result characterizes the unique stationary equilibrium of the game. To ease notations, we introduce two parameters that we use throughout : $\Phi = \frac{h}{1-\beta(1-h)}$ and $\Delta = \frac{\beta(1-h)}{1-\beta(1-h)}$.

Proposition 1 *In the symmetric model, for any $h \in [0, 1]$, there exists a unique stationary equilibrium. There is a threshold $0 < h_1 < 1$, such that:*

- *for $h \in [0, h_1)$, the agreement set A is centered in $c = 1/2$ and has a length $l = \frac{1}{2\Delta}\left(\sqrt{1 + 4\Delta(1 - \Phi D)} - 1\right)$, increasing in the threat h.*
- *for $h \in [h_1, 1]$, the agreement set is $A = [0, 1]$.*

The main take-away of Proposition 1 is that the ability of both agents to compromise, as measured by the (instantaneous) agreement probability l, always increases with the threat h. Intuitively, when the threat h is low enough, both players rationally reject deals that are too far from their bliss point. On the contrary, when the threat reaches h_1, the risk of a hard outcome is so high that both agents prefer to compromise whatever deal is proposed.

By symmetry of the model, the center of the agreement set (i.e. the expected location of an accepted deal) always coincides with $1/2$, the center of the outcome space. The length of the agreement set depends on the severity of the hard outcome, as measured by D. When it becomes more severe (D decreases), agents become more likely to compromise (l increases). As we show in Section 6, all these features of the agreement set are preserved in an extended model where proposed deals are drawn from a symmetric interval that is narrower than X.

To illustrate Proposition 1, we draw the equilibrium agreement set on a first example.

Example 1

We focus on the example where $\beta = 0.95$, $D = -.4$ and $B = 0$. For these parameters, we draw the bounds of the agreement set for all values of h between 0 and 1 on Figure 1.

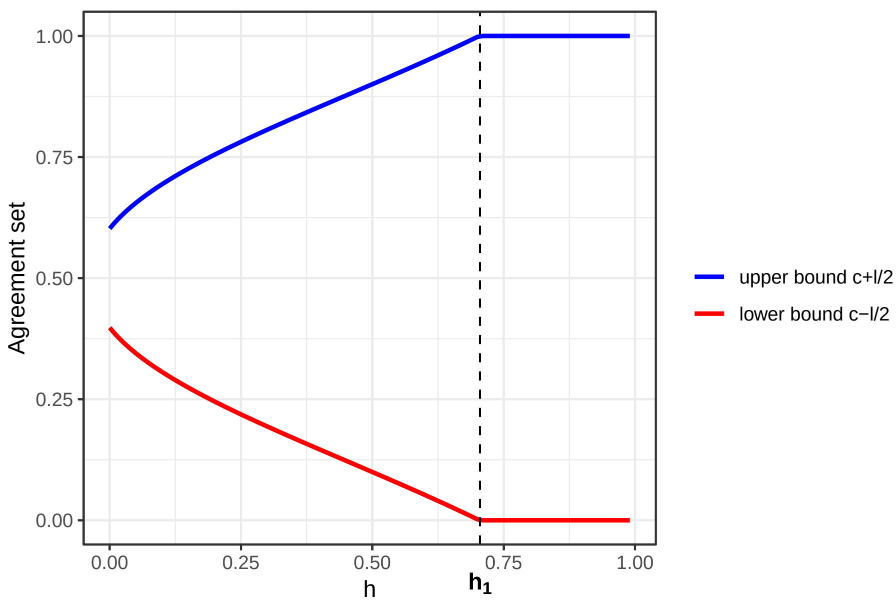

Figure 1: Agreement set in Example 1

We observe two regimes on this picture. For $h \leq h_1 \approx 0.7$, both agents reject some deals at equilibrium. As h increases, the length of the agreement set increases up to $h = h_1$, where both agents accept all deals. As the model is symmetric, no agent is advantaged, and the center is equal to $1/2$.

4.2 Welfare

We now characterize equilibrium welfare in the symmetric model. We denote by W_θ the expected utility of each agent $\theta \in \{0,1\}$ at equilibrium, which satisfies the following recursive equation:

$$W_\theta = \mathbb{P}(x \in A)\mathbb{E}[u_\theta(x) \mid x \in A] + \mathbb{P}(x \notin A)\left(hd_\theta + \beta(1-h)W_\theta\right).$$

In this formula, the welfare is computed at the beginning of a period: either the randomly selected deal x belongs to the agreement set, in which case it yields $\mathbb{E}[u_\theta(x) \mid x \in A]$ in expectation, or it fails to do so and hence, either the hard outcome is selected or a new period starts, with expected utility W_θ. Therefore, the welfare is given by:

$$\forall \theta \in \{0,1\}, \qquad W_\theta = \frac{(1-l)hd_\theta + l\mathbb{E}[u_\theta(x) \mid x \in A]}{1-\beta(1-h)(1-l)}. \tag{2}$$

The following result asserts that this welfare only depends on the length of the agreement set at equilibrium.

Theorem 2 *In the symmetric model, agents' equilibrium welfare solely depends on the length of the agreement set l, and follows a convex function, given by:*

$$W_0 = W_1 = \frac{1}{2}\left(1 - l + l^2\right).$$

As a function of l, welfare is convex, symmetric around $1/2$, and reaches its maximum either at $l = 0$ or $l = 1$.[11] As l is increasing in h, the main lesson of Theorem 2 is that welfare is not monotonic as a function of the threat h. While welfare is maximal when the threat h is high enough to enforce full agreement, this does not mean that decreasing the threat always reduces welfare. On the contrary, when the agreement probability l is below $1/2$, then decreasing the threat increases welfare. The insight is particularly relevant when the party choosing the threat is constrained, for instance if threats above some threshold $\overline{h}$ are deemed non-credible. In such case, the optimal

[11] To see this, note that $W_0 = W_1 = \frac{1-l(1-l)}{2}$.

value of the threat is either to fix a maximal threat $\overline{h}$ or no threat at all ($h = 0$).

To further illustrate the convexity result in Theorem 2, we decompose welfare as the product of two factors: *instant utility* and *delay factor*. The instant utility is the undiscounted expected utility of the eventual outcome. As the agreement set is centered in 1/2, the average utility of an accepted deal is 1/2 for each agent. Thus, the instant utility of an agent $\theta \in \{0, 1\}$ can be written as the convex combination of 1/2 and d_θ, this last term being weighted by the *overall probability of hard outcome* $\mathbb{P}(d \mid A)$. The delay factor is the ratio of welfare to instant utility, it accounts for the delay incurred in reaching an agreement through the bargaining process. Formally, the decomposition can be derived from equation (2) as follows:[12]

$$\begin{aligned} W_\theta &= \frac{(1-l)hd_\theta + l(1/2)}{(1-l)h + l} \times \frac{(1-l)h+l}{1-\beta(1-h)(1-l)} \\ &= \underbrace{[\mathbb{P}(d \mid A)d_\theta + (1-\mathbb{P}(d \mid A))\frac{1}{2}]}_{\text{instant utility}} \times \underbrace{\frac{1-(1-h)(1-l)}{1-\beta(1-h)(1-l)}}_{\text{delay factor}}. \end{aligned}$$

The shape of welfare as a function of the threat h thus depends on the relative importance of those two terms. The delay factor is increasing in the threat h.[13] By contrast, the instant utility displays a U-curve, because the overall probability of hard outcome is hump-shaped, as it is null both when there is no threat and when the threat is high enough to force full agreement. When the discount factor β is small, the (instantaneous) agreement probability l is large (above 1/2) even without threat, and welfare is then only increasing as a function of h by Theorem 2. This arises because, as β is low, the evolution of the delay factor dominates that of the instant utility. When β is large however, the agreement probability l is below 1/2 for a low threat h. In that case, welfare displays a U-curve as a function of h by Theorem 2. This is because, as β is large, the evolution of the instant utility dominates that of the delay factor. This second case corresponds to what arises in Example 1 (where $l \approx 0.204$ when $h = 0$), as illustrated in Figure 2.

[12]To see this, note that $\mathbb{P}(d \mid A) = (1-l)h \sum_{k=0}^{+\infty} (1-l)^k (1-h)^k = \dfrac{(1-l)h}{1-(1-l)(1-h)}$.

[13]Indeed, $(1-h)(1-l)$ decreases with h, while $\frac{1-x}{1-\beta x}$ decreases with x, as $\beta < 1$.

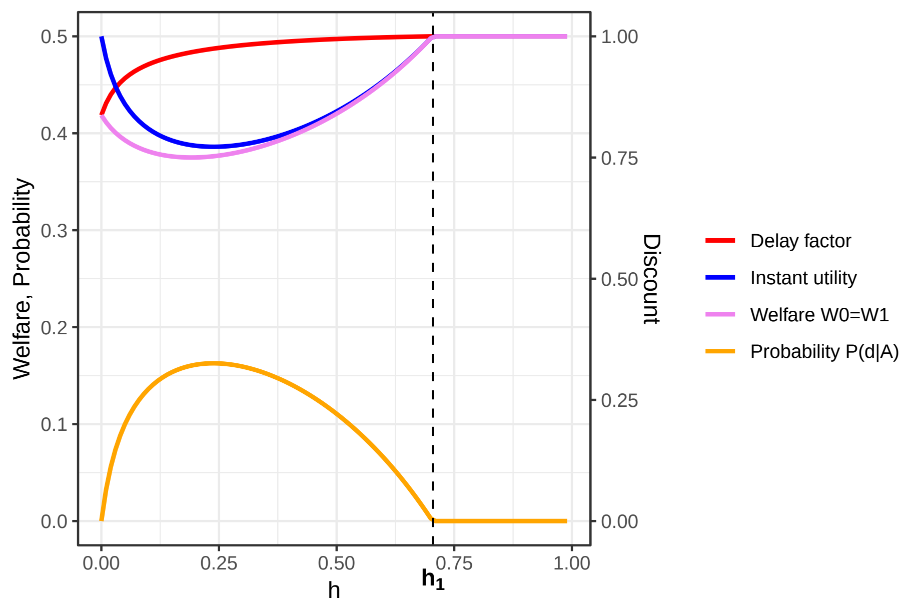

Figure 2: Welfare decomposition in Example 1.

Figure 2 illustrates the previous discussion: agents' welfare displays a U-curve, it is the product of the delay factor, increasing in h, and of the instant utility, which itself follows a U-curve since the probability $\mathbb{P}(d \mid A)$ is hump-shaped.

4.3 Welfare of a third party

We now consider the problem of a third party, noted E (for external), different from agents 0 and 1, who would be choosing the level of threat h. We assume that, as for agents 0 and 1, the utility of the third party u_E is affine and non-negative on $X = [0, 1]$. Moreover, we denote his utility from the outside option by $u_E(d) = d_E \in \mathbb{R}$, and his utility for a deal located at $1/2$ by $v_E = u_E(1/2)$. Note that, since the agreement set A is symmetric, we also have $v_E = \mathbb{E}[u_E(x) \mid x \in A]$.

As before, we may write the third party's welfare as:

$$W_E = \underbrace{[\mathbb{P}(d \mid A) d_E + (1 - \mathbb{P}(d \mid A)) v_E]}_{\text{instant utility}} \times \underbrace{\frac{1 - (1-h)(1-l)}{1 - \beta(1-h)(1-l)}}_{\text{delay factor}}.$$

Using the decomposition, we obtain the following result on the welfare-maximizing threat for the third party.

Proposition 2 *In the symmetric model, there exists a threshold* $\overline{d_E} > v_E$ *such that:*

- *if* $d_E < \overline{d_E}$*, the equilibrium welfare of the third party is maximal for any threat* $h \in [h_1, 1]$*. For this threat level, the hard outcome never occurs in equilibrium.*
- *if* $d_E > \overline{d_E}$*, the equilibrium welfare of the third party is maximal for a threat* $h < h_1$*. For this threat level, the hard outcome occurs with positive probability in equilibrium.*

The result underscores that the third party will choose an interior threat ($h < h_1$) only if he derives a high enough utility from the outside option. The utility threshold $\overline{d_E}$ is higher than the third party's expected utility from a deal v_E. This is due to the delay factor: if the outside option is equivalent to the expected deal ($d_E = v_E$), the third party prefers imposing the maximal threat, to obtain the utility v_E in expectation without any delay. We illustrate on Figure 3 how the welfare-maximizing threat of the third party evolves as a function of d_E in Example 1, for $v_E = 1/2$.

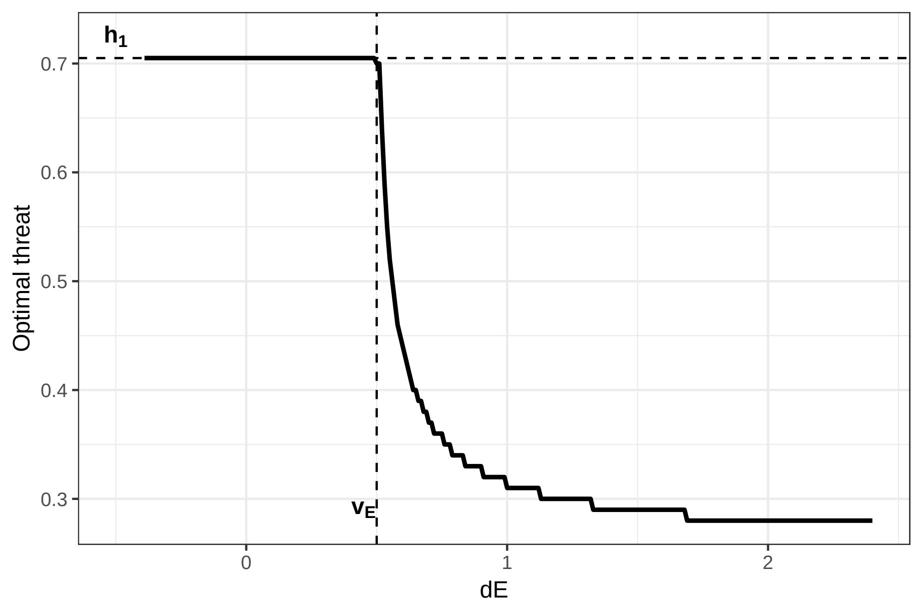

Figure 3: Optimal threat of the third party in Example 1 for $v_E = 1/2$

In this example, the optimal threat of the third party quickly falls below $h_1 \approx 0.7$ when d_E passes above v_E.

5 Asymmetric model

We now consider the asymmetric model where the hard outcome affects both players in a different manner. More precisely, we let $B > 0$ in the rest of the section so that $d_1 > d_0$ making player 1 less affected in the event of a hard outcome. The main difference with the symmetric model is the emergence of a new type of equilibrium, in which agent 0 accepts any deal whereas agent 1 remains selective on the proposals that he accepts.

5.1 Equilibrium

To ease their description, we divide stationary equilibria into types, corresponding to the number of agents that reject some proposed deals at equilibrium. At a *two-*

sided equilibrium, both agents reject some proposals: for any $\theta \in \{0, 1\}$, $w_\theta > 0$. At a *one-sided equilibrium*, only agent 1 does so and hence: $w_0 \leq 0$ and $w_1 > 0$. Finally, both agents accept all deals at a *full-agreement equilibrium*.

Proposition 3 *In the asymmetric model,*

- *at any two-sided equilibrium, the agreement set has center* $c = \frac{1}{2}(1 + \Phi B)$ *and length* $l = \frac{1}{2\Delta}\left(\sqrt{1 + 4\Delta(1 - \Phi D)} - 1\right)$,
- *at any one-sided equilibrium, the agreement set has center* $c = 1 - \frac{l}{2}$ *and length* $l = \frac{1}{\Delta}\left(\sqrt{1 + \Delta(2 - \Phi(B + D))} - 1\right)$,
- *at any full-agreement equilibrium, the agreement set has center* $\frac{1}{2}$ *and length* $l = 1$.

At a two-sided equilibrium, the center of the agreement set only depends on the bias $B = d_1 - d_0$, not on $D = d_1 + d_0$. Conditional on reaching a deal, the expected location of the deal, c, increases with the relative advantage of agent 1 at the hard outcome, B. By contrast, the length of the agreement set does not depend on B, and is thus the same as in the symmetric model studied earlier.

At a one-sided equilibrium, agent 0 accepts any proposed deal. As a result, both the center and the length of the agreement set depend on agent 1's value for the hard outcome $d_1 = (D + B)/2$, but not on agent 0's value for the hard outcome $d_0 = (D - B)/2$. As agent 1's value for the hard outcome increases ($D + B$ increases), the length of the agreement set, l, decreases, as agent 1 becomes less likely to compromise. As a result, in that case, the expected deal c increases, moving closer to agent 1's bliss point.

As in the symmetric setting, when the threat h is low, both players reject some deals at equilibrium. When h is neither too low nor too high, the disadvantaged player is willing to accept any deal to avoid the hard outcome whereas the advantaged player remains selective on which deals to accept, leading to a one-sided equilibrium. Then, when h is high enough, both agents compromise whatever deal is proposed.

Proposition 4 *In the asymmetric model, for any $h \in [0,1]$, there exists a unique stationary equilibrium. There are thresholds $0 < h_1 < h_2 < 1$, such that:*

- *for $h \in [0, h_1)$, the equilibrium is two-sided. In this equilibrium, both l and c are increasing in h.*
- *for $h \in [h_1, h_2)$, the equilibrium is one-sided. In this equilibrium, l is increasing in h and c is decreasing in h.*
- *for $h \in [h_2, 1]$, the equilibrium displays full agreement.*

As in the symmetric model, the agreement probability always increases with the threat h. Moreover, the location of the expected deal, c, varies non-monotonically with h, as it is equal to $1/2$ for both $h = 0$ and $h = 1$. Starting from $h = 0$, the expected deal first increases with h, up to the point h_1 at which agent 0 accepts any deal, and then decreases to reach $1/2$ when $h = h_2$, the point where both agents fully compromise. To illustrate Proposition 4, we draw the equilibrium agreement set as a function of h on a second, asymmetric example.

Example 2

We focus on the example where $\beta = 0.95$, $D = -.4$ and $B = 0.2 > 0$. For these parameters, we draw the bounds and the center of the agreement set for all values of h between 0 and 1 on Figure 4.

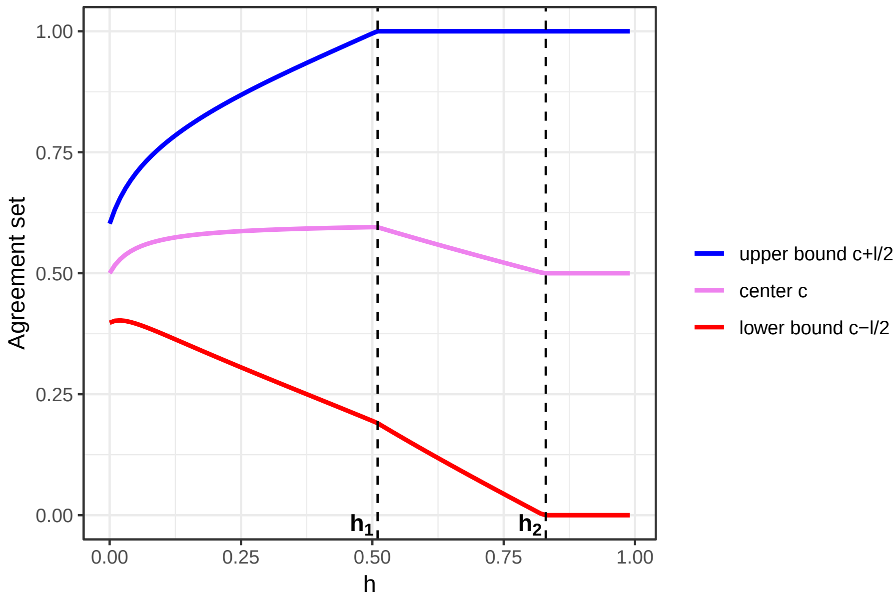

Figure 4: Agreement set in Example 2

We observe three regimes on this picture. For $h \leq h_1 \approx 0.5$, we have a two-sided equilibrium where both agents reject some deals. As the threat h increases, the length of the agreement set increases. Moreover, the center of the agreement set increases as well. This means that the advantage of agent 1 (in terms of the expected deal) becomes stronger when the hard outcome becomes more likely.

For $h \geq h_1 \approx 0.5$ and $h \leq h_2 \approx 0.8$, we enter into a second regime where only agent 1 rejects some deals, while agents 0 accepts all. In this regime, it remains true that l increases with h, i.e. that both agents become more willing to compromise. However, as agent 0 already accepts all deals, all this compromise effort is borne by agent 1, and the expected deal becomes closer to $1/2$ as h increases.

Finally, when h_2 is reached, both agents accept all deals, and full agreement remains for any further increase in the threat h.

5.2 Welfare

In this section, we characterize the welfare of each agent in the asymmetric model for each type of equilibrium. First, note that any deal is accepted at a full agreement equilibrium, so that $W_0 = W_1 = 1/2$.

We then consider the welfare associated to two-sided equilibria.

Proposition 5 *At any two-sided equilibrium w, agents' welfare is given by:*

$$W_0 = \frac{1 - l + l^2}{2} - \frac{B\Phi}{2} \qquad \textit{and} \qquad W_1 = \frac{1 - l + l^2}{2} + \frac{B\Phi}{2},$$

where l denotes the length of the agreement set associated to w.

The main observation of Proposition 5 is that agents' welfare at a two-sided equilibrium can be split in two parts. First, a common-value part is the average welfare, given by $\overline{W} = \frac{1-l+l^2}{2}$, as for the symmetric model. To obtain an agent's welfare, one needs to add a second, zero-sum and private-value part, of magnitude $\frac{B\Phi}{2} = c - 1/2$, so that we have $W_0 = \overline{W} - (c - 1/2)$ and $W_1 = \overline{W} + (c - 1/2)$. When the agreement set is centered in $\frac{1}{2}$, both players share the same welfare since the zero-sum part vanishes. If the center is closer to agent 1's bliss point, agent 1's welfare increases by the same amount as agent 0's welfare decreases.

The following result focuses on one-sided equilibrium welfare.

Proposition 6 *At any one-sided equilibrium w, agents' welfare is such that:*

$$W_1 = 1 - l + \frac{l^2}{2} > 1/2 \qquad \textit{and} \qquad W_0 < 1 - W_1 < 1/2,$$

where l denotes the length of the agreement set associated to w.

We observe that agent 1's welfare decreases with the agreement probability l, and thus with h, at a one-sided equilibrium. Nevertheless, agent 1 always remains advantaged, in the sense that he achieves a welfare higher than $1/2$, the welfare level reached under full agreement. By contrast, the welfare of agent 0 remains below $1/2$.

5.3 Welfare-maximizing threats

In this section, we describe how each agent would choose the level of threat h, if he were to choose it unilaterally.

Proposition 7 *In the asymmetric model,*

- *the equilibrium welfare of agent* 1 *is maximal for a threat* $h \in (0, h_1]$. *For this threat level,* $W_1 > 1/2$ *and the hard outcome occurs with positive probability in equilibrium.*
- *the equilibrium welfare of agent* 0 *is maximal for any threat* $h \in [h_2, 1]$. *For this threat level,* $W_0 = 1/2$ *and the hard outcome never occurs in equilibrium.*

We observe in Proposition 7 a discrepancy between the two agents. Agent 0 would always choose a threat h high enough to enforce full agreement, so that the hard outcome never occurs. This is not the case for agent 1, who would always choose an intermediate threat $h \in (0, h_1]$. This means that agent 1 prefers to use the threat at his advantage, at the risk of seeing the hard outcome occurring. Indeed, for such threat h, the hard outcome arises on-path with positive probability.

To illustrate the optimal choice of threats, we depict on Figure 5 the welfare of each agent as a function of the threat h in Example 2.

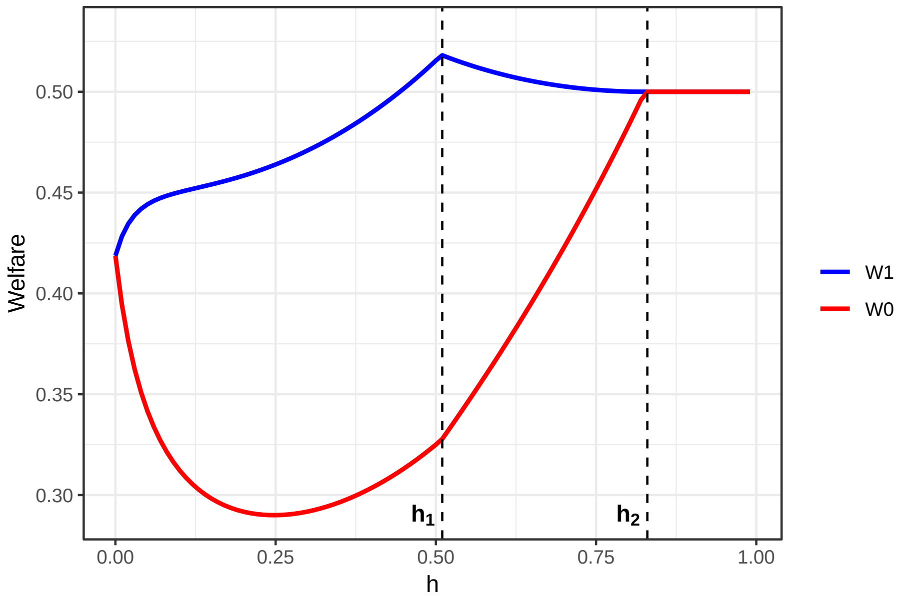

Figure 5: Agents' welfare in Example 2

In this example, agent 0's welfare displays a U-curve, as in the symmetric model, and is maximal for $h \geq h_2 \approx 0.8$, when agents reach full agreement. However, the welfare of agent 1 is maximized for a threat $h = h_1 \approx 0.5$. This means that agent 1 would choose a threat just high enough to make his opponent accept every deal.[14]

6 Restricted deals

We consider an extension where the set of deals is $X = [1 - b, b]$ for $b \in (1/2, 1]$, so that the previous model corresponds to the particular case $b = 1$. A stationary equilibrium remains characterized by an agreement set $A \subseteq X$ satisfying equation (1). For an agreement set A of center $c \in X$ and length $l \in (0, 2b - 1]$, we denote by $\lambda = \frac{l}{2b-1}$ the probability that a deal is drawn in the agreement set A. The following result extends Theorem 1, Proposition 1 and Proposition 3.

[14]We note that this is not a general result: for the same parameter values but $\beta = 0.999$, W_1 is maximal for $h < h_1$.

Proposition 8 *In the extended model, there is a unique stationary equilibrium. There are thresholds* $h_1, h_2 \in (0,1)$ *with* $h_1 \leq h_2$ *such that:*

- *for* $h < h_1$*, the equilibrium is two-sided, i.e.* $w_0, w_1 > 1-b$*. In this equilibrium,* $c = (1+\Phi B)/2$ *and*

$$\lambda = \frac{1}{2\Delta}\left(\sqrt{1+4\Delta\left(\frac{1-\Phi D}{2b-1}\right)}-1\right)$$

- *for* $h_1 \leq h < h_2$*, the equilibrium is one-sided, i.e.* $w_1 > 1-b \geq w_0$*. In this equilibrium,* $c = b-(2b-1)\lambda$ *and*

$$\lambda = \frac{1}{\Delta}\left(\sqrt{1+\Delta\left(\frac{2b-\Phi(B+D)}{2b-1}\right)}-1\right)$$

- *for* $h \geq h_2$*, the equilibrium displays full agreement, i.e.* $c = 1/2$ *and* $\lambda = 1$*.*

For each equilibrium regime, the probability of reaching a deal, λ*, decreases with* b*. Moreover, the thresholds* h_1 *and* h_2 *are both increasing with* b*.*

The main insights of the model remain true in the extension to restricted deals. Furthermore, restricting deals entail more compromise in each equilibrium regime, and each agent becomes more likely to fully compromise (i.e. w_θ reaching $1-b$) when deals are more restricted. We illustrate these results for Example 2, for $b = 0.8$ and $b = 1$, on Figure 6.

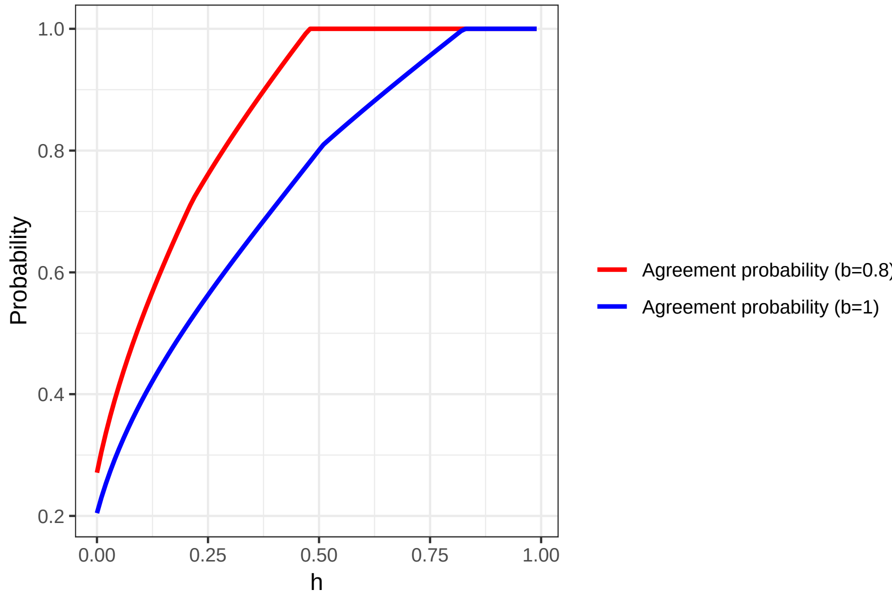

Figure 6: Agreement probability in Example 2, for $b = 0.8$ and $b = 1$

Finally, we note that the central result on welfare in the symmetric model exhibited in Theorem 2 extends as well.

Theorem 3 *In the symmetric extended model, agents' equilibrium welfare solely depends on b and on the agreement probability λ, and follows a convex function, given by:*

$$W_0 = W_1 = \frac{1}{2} + \frac{2b-1}{2}\left(-\lambda + \lambda^2\right).$$

As before, agents' welfare is convex as a function of the agreement probability λ, reaching its minimum for $\lambda = 1/2$.

7 Conclusions

The broad question of how threats during a negotiation may alter actual and potential outcomes is central to many political and non political negotiations. We tried to analyze how these threats can affect the negotiation gridlock, welfare of the negotiating parties and finally when they can be used strategically by either party, or third parties, to their advantage. Our model is extendable to more general distributions of potential agreements and to asymmetric information in which the payoffs to hard outcomes are private information of the two sides. Namely, the results of our model may be used as the last stage of a broader model which endogenizes the credibility walk-away threat announcements.

While not walk-away threats, similar strategic threats and political brinkmanship have been at the core of several major US political crises and government shutdowns. For instance, the longest U.S. government shutdown in history (from December 22, 2018, for 35 days) occurred when the US Congress and President Donald Trump could not agree on an appropriations bill to fund the operations of the federal government for the 2019 fiscal year, or a temporary continuing resolution that would extend the deadline for passing a bill. The shutdown stemmed from an impasse over Trump's demand for federal funds for a U.S.–Mexico border wall. Similar brinkmanship tactics were at the core of the US debt ceiling crises (Obama (2011 and 2013), Clinton 1995, both facing a republican congress).[15] For instance, in 2013: the Republican Party in Congress refused to raise the debt ceiling unless President Obama would have defunded the Affordable Care Act (Obamacare), his signature legislative achievement. The US Treasury stated that it would have to *delay payments* if funds could not be raised through these measures: the US defaulting on its debt became more likely as days passed without an agreement and would have resulted in permanent damage to the economy.[16] A similar Debt Ceiling episode happened in 2011.[17]

[15]These are designed to provide extra pressure on the counterparts. For instance, on January 2013, Paul Ryan, Chairman of the House Budget Committee argued that giving Treasury enough borrowing power to postpone default until mid-March would allow Republicans to gain an advantage over Obama and Democrats in debt ceiling negotiations.

[16]Treasury Secretary Timothy Geithner warned that "failure to raise the limit would precipitate a default by the United States. Default would effectively impose a significant and long-lasting tax on all Americans and all American businesses and could lead to the loss of millions of American jobs. Even a very short-term or limited default would have catastrophic economic consequences that would last for decades."

[17]As in the subsequent 2013 episode, U.S. government debt was downgraded (for the first time in its history), the stock market fell, measures of volatility jumped, and credit risk spreads widened noticeably.

A Proofs

A.1 Proof of Theorem 1

Proof. For $\theta \in \{0,1\}$, let ψ_θ be the function defined on $\mathbb{R}^2$ by:

$$\psi_\theta(w) = hd_\theta + \beta(1-h)\mathbb{P}(x \in A_w)\mathbb{E}[u_\theta(x) \mid x \in A_w] + \beta(1-h)\mathbb{P}(x \notin A_w)w_\theta.$$

We have that for any $w \in (-\infty, 1]^2$,

$$hd_\theta + \beta(1-h)w_\theta \leq \psi_\theta(w) \leq hd_\theta + \beta(1-h).$$

Let $w_\theta^{\max} = hd_\theta + \beta(1-h) \leq 1$ and $w_\theta^{\min} = \frac{hd_\theta}{1-\beta(1-h)}$. As we assumed $d_\theta \leq 0$, we obtain that $w_\theta^{\max} \geq w_\theta^{\min}$. Now, for $(w_0, w_1) \in [w_0^{\min}, w_0^{\max}] \times [w_1^{\min}, w_1^{\max}]$, we have for any $\theta \in \{0,1\}$, $\psi_\theta(w_\theta) \leq w_\theta^{\max}$ and:

$$\begin{aligned}
\psi_\theta(w_\theta) &\geq hd_\theta + \beta(1-h)w_\theta^{\min} \\
&\geq hd_\theta + \beta\delta\frac{hd_\theta}{1-\beta(1-h)} \\
&\geq \frac{hd_\theta}{1-\beta(1-h)} = w_\theta^{\min}.
\end{aligned}$$

Hence, the application $\psi : [w_0^{\min}, w_0^{\max}] \times [w_1^{\min}, w_1^{\max}] \to [w_0^{\min}, w_0^{\max}] \times [w_1^{\min}, w_1^{\max}]$ defined by $\psi(w) = (\psi_0(w), \psi_1(w))$ is continuous, from a non-empty convex compact set onto itself, so it admits a fixed point w^* by Brouwer's theorem. Hence, the game admits w^* as a stationary equilibrium.

At a stationary equilibrium w, the agreement set can be written as

$$A = \{x \in X \mid 1 - x \geq w_0 \text{ and } x \geq w_1\} = \{x \in X \mid w_1 \leq x \leq 1 - w_0\}.$$

If the agreement set A was empty, we would have by application of equation (1), $w_\theta = hd_\theta/(1-\beta(1-h)) \leq 0$. In that case, we would have $X \subseteq [w_1, 1-w_0]$, a contradiction with A being empty.

Thus, A is a non-empty, closed interval. Let c be A's center and assume that

$c < 1/2$. In that case, we have

$$\mathbb{E}[u_0(x) \mid x \in A] > 1/2 > \mathbb{E}[u_1(x) \mid x \in A].$$

We obtain:

$$\begin{aligned} w_0 &= \frac{hD + \beta(1-h)\mathbb{P}(x \in A)\mathbb{E}[u_0(x) \mid x \in A]}{1 - \beta(1-h)\mathbb{P}(x \notin A)} \\ &\leq \frac{hD + \beta(1-h)\mathbb{P}(x \in A)(1/2)}{1 - \beta(1-h)\mathbb{P}(x \notin A)} \\ &\leq \frac{hD + \beta(1-h)\mathbb{P}(x \in A)\mathbb{E}[u_1(x) \mid x \in A]}{1 - \beta(1-h)\mathbb{P}(x \notin A)} \leq w_1. \end{aligned}$$

This contradicts that $A = \{x \in [0,1] \mid w_1 \leq x \leq 1 - w_0\}$ is centered in $c < 1/2$. ■

A.2 Proof of Proposition 1, Proposition 3 and Proposition 4

Proof. Since Proposition 1 deals with the symmetric model ($B = 0$), and Proposition 3 and Proposition 4 deal with the asymmetric one ($B > 0$), we give directly the proof for the general case ($B \geq 0$).

A. Agreement sets in equilibrium

Two-sided equilibrium. Let w be a two-sided equilibrium and let $A = [c - l/2, c + l/2]$ be its agreement set. As proposals are uniformly drawn from $[0,1]$, we have $\mathbb{P}(x \in A) = l$. The expected utilities of a deal in the agreement set are given by:

$$\mathbb{E}[u_0(x) \mid x \in A] = \int_{c-\frac{l}{2}}^{c+\frac{l}{2}} (1-x)\frac{1}{l}dx = (1-c), \quad \mathbb{E}[u_1(x) \mid x \in A] = \int_{c-\frac{l}{2}}^{c+\frac{l}{2}} x\frac{1}{l}dx = c.$$

The reservation values are thus given by:

$$w_0 = \frac{hd_0 + \beta(1-h)l(1-c)}{1 - \beta(1-h) + \beta(1-h)l} = \frac{\Phi d_0 + \Delta l(1-c)}{1 + \Delta l}$$

and

$$w_1 = \frac{hd_1 + \beta(1-h)lc}{1 - \beta(1-h) + \beta(1-h)l} = \frac{\Phi d_1 + \Delta lc}{1 + \Delta l}.$$

The agreement set is $A = [c - l/2, c + l/2] = \{x \in [0,1] \mid w_1 \leq x \leq 1 - w_0\}$. We obtain

$$\begin{cases} l &= 1 - (w_0 + w_1) \\ c &= \frac{1+w_1-w_0}{2} \end{cases}$$

Solving for c first, we get:

$$2c = 1 + \frac{\Phi B + \Delta l\,(2c-1)}{1 + \Delta l} \quad \Leftrightarrow \quad c = \frac{1 + \Phi B}{2}.$$

Solving for l, we obtain:

$$\begin{aligned} 1 - l = \frac{\Phi D + \Delta l}{1 + \Delta l} &\Leftrightarrow \quad 1 - \Phi D = l + \Delta l^2 \\ &\Leftrightarrow \quad l = \frac{1}{2\Delta}\left(-1 + \sqrt{1 + 4\Delta(1 - \Phi D)}\right). \end{aligned}$$

One-sided equilibrium. Let w be a one-sided equilibrium and let A be its agreement set. We may write:

$$w_1 = \frac{\Phi d_1 + \Delta l c}{1 + \Delta l}.$$

Moreover, the agreement set is $A = [w_1, 1]$ by assumption, so that $l = 1 - w_1$ and $c = 1 - \frac{l}{2}$. We obtain:

$$(1 - l)(1 + \Delta l) = \Phi d_1 + \Delta l(1 - l/2) \quad \Leftrightarrow \quad 1 - \Phi d_1 = l + \Delta l^2/2.$$

Hence, we get:

$$l \;=\; \frac{1}{\Delta}\left(\sqrt{1 + 2\Delta(1 - \Phi d_1)} - 1\right).$$

B. Conditions for existence

We first provide conditions for the existence of each type of equilibrium. Then, we partition the set of h values corresponding to each type of equilibrium. Finally, we show the comparative statics results.

Two-sided equilibria: existence. Two-sided equilibria are characterized by the system of equations: $c = \frac{1+\Phi B}{2}$ and $1 - \Phi D = l + \Delta l^2$. As $\frac{1+\Phi B}{2} \geq 1/2$, a necessary and sufficient condition for such an equilibrium to exist is that the previous system admits a solution with $c+l/2 \leq 1$, i.e. $1-\Phi B-l > 0$. Hence, a two-sided equilibrium exists if and only if:

$$\exists l < 1 - \Phi B, \qquad 1 - \Phi D = l + \Delta l^2.$$

This condition is equivalent to $1 - \Phi D < (1 - \Phi B) + \Delta(1 - \Phi B)^2$, as $l + \Delta l^2$ is increasing and continuous on $[0, 1 - \Phi B)$. Thus, a two-sided equilibrium exists if and only if:

$$\Phi(B - D) < \Delta(1 - \Phi B)^2. \tag{3}$$

One-sided equilibria: existence. One-sided equilibria are characterized by the equation $1 - \Phi(B + D)/2 = l + \Delta l^2/2$. Such an equilibrium exists if and only if $w_0 \leq 0$ and $l < 1$. As $w_1 = 1 - l$, we may write, for such an equilibrium:

$$w_0 = (w_0 + w_1) - w_1 = \frac{\Phi D + \Delta l}{1 + \Delta l} - (1 - l) = \frac{\Phi D + \Delta l - (1 - l)(1 + \Delta l)}{1 + \Delta l}.$$

Hence, we can write:

$$\begin{aligned}
w_0 \leq 0 \quad &\Leftrightarrow \quad \Phi D + \Delta l \leq (1 - l)(1 + \Delta l) \\
&\Leftrightarrow \quad \Phi D + \Delta l \leq 1 - l + \Delta l - \Delta l^2 \\
&\Leftrightarrow \quad l + \Delta l^2 \leq 1 - \Phi D \\
&\Leftrightarrow \quad 2(l + \Delta l^2/2) - l \leq 1 - \Phi D \\
&\Leftrightarrow \quad 2(1 - \Phi(B + D)/2) - l \leq 1 - \Phi D \\
&\Leftrightarrow \quad 1 - \Phi B \leq l.
\end{aligned}$$

To conclude, a one-sided equilibrium exists if and only if:

$$\exists l \in [1 - \Phi B, 1), \qquad 1 - \Phi(B + D)/2 = l + \Delta l^2/2.$$

This condition is equivalent to $1+\Delta/2 > 1-\Phi(B+D)/2 \geq (1-\Phi B)+\Delta(1-\Phi B)^2/2$, as $l+\Delta l^2/2$ is increasing and continuous on $[1-\Phi B, 1]$. Thus, a one-sided equilibrium exists if and only if:

$$\Phi(B+D)+\Delta > 0 \quad \text{and} \quad \Phi(B-D) \geq \Delta(1-\Phi B)^2. \tag{4}$$

Full-agreement equilibria: existence. In a full agreement equilibrium, we have $c = 1/2$ and $l = 1$. As $B \geq 0$, we have $w_1 \geq w_0$, and a necessary and sufficient condition for existence is $w_1 \leq 0$. This can be written $w_1 = (\Phi(B+D)/2 + \Delta(1/2))/(1+\Delta) \leq 0$. Thus, a full-agreement equilibrium exists if and only if:

$$\Phi(B+D)+\Delta \leq 0. \tag{5}$$

C. Equilibrium uniqueness.

We consider the equations $\Phi(B-D) = \Delta(1-\Phi B)^2$ and $\Phi(B+D)+\Delta = 0$. As functions of h, Φ is increasing, with $\Phi(h=0) = 0$ and Δ is decreasing with $\Delta(h=1) = 0$. Thus, the two equations above each have a unique solution, that we denote respectively by $h_1 \in (0,1)$ and by $h_2 \in (0,1)$. We observe that:

$$\frac{\Phi}{\Delta}(h_2) = \frac{1}{-B-D} \geq \frac{(1-\Phi(h_1)B)^2}{B-D} = \frac{\Phi}{\Delta}(h_1).$$

As Φ/Δ is increasing as a function of h, we obtain that $h_1 \leq h_2$, with an equality if and only if $B = 0$. To conclude, using the conditions (3), (4) and (5), we obtain that for any $h \in [0,1]$, there exists a unique equilibrium:

- if $h < h_1$, there is a two-sided equilibrium (only (3) can be satisfied)
- if $h_2 \leq h < h_2$, there is a one-sided equilibrium (only (4) can be satisfied)
- if $h \geq h_2$, there is a full-agreement equilibrium (only (5) can be satisfied).

Two-sided equilibria: comparative statics. It is immediate that c is a non-decreasing function of h, strictly increasing whenever $B > 0$.

The length l is obtained as the solution of the equation $1 - D\Phi = l + \Delta l^2$. As shown on Figure 7, l must increase as h increases.

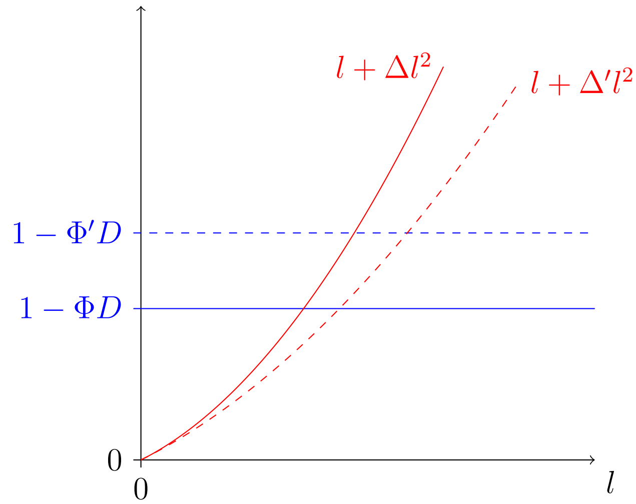

Figure 7: Characterization of l (h increases)

One-sided equilibria: comparative statics. The length l is obtained as the solution of the equation: $1 - \Phi d_1 = l + \Delta l^2/2$. As $d_1 \leq 0$, we may apply the same argument as for the two-sided equilibria, depicted on Figure 7. We obtain that l increases with h, and as a result, $c = 1 - l/2$ decreases with h. ■

A.3 Proof of Theorem 2

Proof. As $B = 0$, we have either full-agreement or a two-sided equilibrium. For a full-agreement equilibrium, we know that the first proposed deal will be accepted with probability one, so that $W_0 = W_1$. As we also have $l = 1$, the formula $W_0 = W_1 = (1 - l + l^2)/2$ is valid.

Let w be a two-sided equilibrium and let A be its agreement set. Noting $l = \mathbb{P}(x \in A)$, we know that:

$$\begin{aligned} W_\theta &= \frac{(1-l)\, hd_\theta + l\mathbb{E}[u_\theta(x) \mid x \in A]}{1 - \beta(1-h)\,(1-l)}, \\ w_\theta &= \frac{hd_\theta + \beta(1-h) l\mathbb{E}[u_\theta(x) \mid x \in A]}{1 - \beta(1-h)\,(1-l)}. \end{aligned}$$

Solving for $l\mathbb{E}[u_\theta(x) \mid x \in A]$ in both expressions w_θ and W_θ we have:

$$\begin{aligned}\frac{l\mathbb{E}[u_\theta(x) \mid x \in A]}{1-\beta(1-h)\,(1-l)} &= W_\theta - \frac{(1-l)\,h}{1-\beta(1-h)\,(1-l)} d_\theta \\ &= \frac{1}{\beta(1-h)} \left(w_\theta - \frac{h}{1-\beta(1-h)\,(1-l)} d_\theta \right).\end{aligned}$$

Thus, simplifying, we have the affine relation:

$$W_\theta = \frac{1}{\beta(1-h)} \left(w_\theta - h d_\theta \right).$$

As w is two-sided, we have $w_0 = w_1 = \frac{1-l}{2}$, and we may write

$$W_0 = W_1 = \frac{(1-l) - hD}{2\beta(1-h)}.$$

As w is two-sided, we know that l is the solution of:

$$\begin{aligned}1 - \Phi D = l + \Delta l^2 \quad &\Leftrightarrow \quad 1 - \frac{h}{1-\beta(1-h)} D = l + \frac{\beta(1-h)}{1-\beta(1-h)} l^2 \\ &\Leftrightarrow \quad 1 - \beta(1-h) - hD = (1-\beta(1-h))\, l + \beta(1-h) l^2 \\ &\Leftrightarrow \quad (1-l) - hD = \beta(1-h) \left(1 - l + l^2\right).\end{aligned}$$

We thus obtain $W_0 = W_1 = (1 - l + l^2)/2$, as desired. ■

A.4 Proof of Proposition 2

Proof. We may write:

$$W_E(d_E, h) = (v_E + P(h)(d_E - v_E)) \times f(h)$$

where the hard outcome probability $P(h) = \mathbb{P}(d \mid A)$ is such that $P(h) > 0 \Leftrightarrow 0 < h < h_1$ and the delay factor $f(h)$ is positive and non-decreasing with h.

As $W_E(d_E, \cdot)$ is constant on $[h_1, 1]$, we study its maximum on $[0, h_1]$. As $f(h_1) > f(0)$, there are two possible cases: either the maximum is reached for $h < h_1$ or for

$h = h_1$.

Case 1. If $W_E(d_E, h) \geq W_E(d_E, h_1)$ for some $h < h_1$, then we have for any $d'_E > d_E$, $W_E(d'_E, h) > W_E(d_E, h) \geq W_E(d_E, h_1) = W_E(d'_E, h_1)$.

Case 2. If $\forall h \in (0, h_1), W_E(d_E, h) \leq W_E(d_E, h_1)$, then we have for any $d'_E < d_E$ and $h \in (0, h_1)$, $W_E(d'_E, h) < W_E(d_E, h) \leq W_E(d_E, h_1) = W_E(d'_E, h_1)$.

We know that Case 2 arises for $d_E \leq v_E$. Thus, there is a critical value for d_E, denoted by $\overline{d_E} \in [v_E, +\infty]$, above which W_E is maximal for $h < h_1$ and below which W_E is maximal for $h = h_1$.

To show that $\overline{d_E} < +\infty$, we prove that Case 1 arises for some d_E large enough. For this, it suffices to show that the function $F(h) := P(h)g(h)$ has a negative derivative in h_1. We may write:

$$F(h) = \frac{(1-l)h}{(1-l)h + l} \times \frac{(1-l)h + l}{1 - \beta(1-l)(1-h)} = \frac{(1-l)h}{1 - \beta(1-l)(1-h)}$$

As $l(h_1) = 1$, we obtain that $\frac{\partial F}{\partial h}(h_1) = -h_1 \frac{\partial l}{\partial h}(h_1) < 0$. The last inequality come from the fact that $\frac{\partial \Delta}{\partial h}(h_1) \neq 0$ and $\frac{\partial \Phi}{\partial h}(h_1) \neq 0$, which imply $\frac{\partial l}{\partial h}(h_1) > 0$, as shown in the proof of Proposition 1.

To show that $\overline{d_E} > v_E$, let us prove that $f'(h_1) > 0$. The delay factor can be written as $f(h) = G((1-l)(1-h))$ where $G(x) = \frac{1-x}{1-\beta x}$. We obtain $G'(x) = \frac{\beta - 1}{(1-\beta x)^2}$. Now, using the fact that $l(h_1) = 1$, we may write:

$$\begin{aligned} f'(h_1) &= G'\left((1 - l(h_1))(1-h_1)\right)\left((1-h_1)\left(-\frac{\partial l}{\partial h}(h_1)\right) - (1 - l(h_1))\right) \\ &= (1-\beta)(1-h_1)\frac{\partial l}{\partial h}(h_1) > 0. \end{aligned}$$

This concludes the proof. ■

A.5 Proof of Proposition 5

Proof. Let w be a two-sided equilibrium and let A be its agreement set. As w is two-sided, we have $w_0 + w_1 = 1 - (c + l/2) + (c - l/2) = 1 - l$. Then, using the same formulas as in the proof of Theorem 2, we may write:

$$W_0 + W_1 = \frac{(1-l) - hD}{\beta(1-h)} = 1 - l + l^2.$$

As w is two-sided, we have $w_1 - w_0 = c - l/2 - (1-(c+l/2)) = 2c - 1 = \Phi B$. Using the formulas from the proof of Theorem 2, we may write:

$$\begin{aligned} W_1 - W_0 &= \frac{1}{\beta(1-h)}\left(w_1 - w_0 - h(d_1 - d_0)\right) \\ &= \frac{1}{\beta(1-h)}\left(\Phi B - hB\right) = \frac{hB}{\beta(1-h)}\left(\frac{1}{1-\beta(1-h)} - 1\right) = \Phi B. \end{aligned}$$

To conclude, we write $W_0 = \frac{W_0 + W_1}{2} - \frac{W_1 - W_0}{2}$ and $W_1 = \frac{W_0 + W_1}{2} + \frac{W_1 - W_0}{2}$, and we obtain the desired formulas. ■

A.6 Proof of Proposition 6

Proof. Let w be a one-sided equilibrium and let A be its agreement set. The length l is the solution of:

$$1 - \Phi d_1 = l + \Delta l^2/2 \quad \Leftrightarrow \quad 1 - l - hd_1 = \beta(1-h)(1 - l + l^2/2).$$

Using the formula for W_θ as a function of w_θ derived in the proof of Theorem 2, we obtain:

$$W_1 = \frac{1}{\beta(1-h)}\left(w_1 - hd_1\right) = \frac{1}{\beta(1-h)}\left(1 - l - hd_1\right) = 1 - l + l^2/2.$$

As $l < 1$, we have that $W_1 > 1/2$.

Then, we may write, using the decomposition of total welfare in instant payoff and delay factor:

$$W_0 + W_1 = \frac{l + (1-l)hD}{1 - \beta(1-h)(1-l)} = \frac{l + (1-l)hD}{l + (1-l)h} \times \frac{1 - (1-l)(1-h)}{1 - \beta(1-h)(1-l)} < 1.$$

Thus $W_0 < 1 - W_1 < 1/2$. ■

A.7 Proof of Proposition 7

Proof. For agent 1, the welfare W_1 is constant, equal to $1/2$ on $[h_2, 1]$. Moreover, W_1 is decreasing on $[h_1, h_2]$, as we know that l is increasing with h in this regime (by Proposition 4) and that $W_1 = 1 - l + l^2/2$ is a decreasing function of l (by Proposition 6). For $h = 0$, we have $W_0 = W_1$ by symmetry, and we know from the proof of Proposition 6 that $W_0 + W_1 < 1$ whenever $l < 1$, we thus have $W_1 < 1/2$. We thus obtained that W_1 is maximal for some $h \in (0, h_1]$.

For agent 0, we have $W_0 \leq W_1$ (since by Theorem 1, $c \geq 1/2$) and we know that whenever $l < 1$, we have $W_0 + W_1 < 1$. Hence, for $l < 1$, $W_0 < 1/2$. Thus, W_0 is maximal for $l = 1$, i.e. $h \in [h_2, 1]$. ■

A.8 Proof of Proposition 8

Proof. The existence of an equilibrium follows from the proof of Theorem 1. As for the proof of Proposition 1, Proposition 3 and Proposition 4, we examine in turn each type of stationary equilibrium.

Two-sided equilibria. At a two-sided equilibrium, we have $1 - b < w_1 < 1 - w_0 < b$, where:

$$w_0 = \frac{\Phi d_0 + \Delta\lambda(1-c)}{1+\Delta\lambda}, \qquad w_1 = \frac{\Phi d_1 + \Delta\lambda c}{1+\Delta\lambda},$$

with $l = 1 - w_1 - w_0 = (2b-1)\lambda$ and $c = \frac{1+w_1-w_0}{2}$. We obtain as before $c = (1+\Phi B)/2$, while λ is given by:

$$1 - (2b-1)\lambda = \frac{\Phi D + \Delta\lambda}{1+\Delta\lambda} \quad \Leftrightarrow \quad \frac{1-\Phi D}{2b-1} = \lambda + \Delta\lambda^2.$$

Hence, the formula for λ written in the proposition. The condition for such an equilibrium to exist can be written as:

$$c + l/2 < b \quad \Leftrightarrow \quad 1 + \Phi B + (2b-1)\lambda < 2b \quad \Leftrightarrow \quad \lambda < 1 - \frac{\Phi B}{2b-1}.$$

To sum-up, a two-sided equilibrium exits if and only if:

$$\exists \lambda < 1 - \frac{\Phi B}{2b-1}, \qquad \frac{1-\Phi D}{2b-1} = \lambda + \Delta\lambda^2.$$

This condition can be written:

$$\frac{1-\Phi D}{2b-1} < \left(1 - \frac{\Phi B}{2b-1}\right) + \Delta \left(1 - \frac{\Phi B}{2b-1}\right)^2,$$

and is equivalent to:

$$\frac{2(1-b)+\Phi(B-D)}{2b-1} < \Delta \left(1 - \frac{\Phi B}{2b-1}\right)^2. \tag{6}$$

One-sided equilibria. At a one-sided equilibrium, we have $1-b < w_1 < b \leq 1-w_0$, where:

$$w_0 = \frac{\Phi d_0 + \Delta\lambda(1-c)}{1+\Delta\lambda}, \qquad w_1 = \frac{\Phi d_1 + \Delta\lambda c}{1+\Delta\lambda},$$

with $l = b - w_1 = (2b-1)\lambda$ and $c = b - l/2 = b(1-\lambda)+\lambda/2$. We obtain, using the expression for w_1:

$$(b - \lambda(2b-1))\,(1+\Delta\lambda) = \Phi d_1 + \Delta\lambda\,(b(1-\lambda)+\lambda/2) \quad \Leftrightarrow \quad \frac{b-\Phi d_1}{2b-1} = \lambda + \Delta\lambda^2/2.$$

Hence, the formula for λ written in the proposition. The conditions for such an equilibrium to exist are $w_1 > 1-b$ and $w_0 \leq 1-b$. The first condition can be written $\lambda < 1$. Writing $w_0 = (w_1+w_0) - w_1 = \frac{\Phi D + \Delta\lambda}{1+\Delta\lambda} - (b-(2b-1)\lambda)$, the condition $w_0 \leq 1-b$ can be written as: $\Phi D + \Delta\lambda - (b-(2b-1)\lambda)(1+\Delta\lambda) \leq (1-b)(1+\Delta l)$, which is equivalent to:

$$\begin{aligned}
\frac{1-\Phi D}{2b-1} \geq \lambda(1+\Delta\lambda) \Leftrightarrow & \quad \frac{1-\Phi D}{2b-1} \geq 2(\lambda + \Delta\lambda^2/2) - \lambda \\
\Leftrightarrow & \quad \frac{1-\Phi D}{2b-1} \geq 2\frac{b-\Phi d_1}{2b-1} - \lambda \\
\Leftrightarrow & \quad \lambda \geq 1 - \frac{\Phi B}{2b-1}.
\end{aligned}$$

To sum-up, a one-sided equilibrium exists if and only if:

$$\exists \lambda \in \left[1 - \frac{\Phi B}{2b-1}, 1\right), \qquad \frac{b - \Phi d_1}{2b-1} = \lambda + \Delta\lambda^2/2.$$

This condition can be written:

$$1 + \Delta/2 > \frac{b - \Phi d_1}{2b-1} \geq \left(1 - \frac{\Phi B}{2b-1}\right) + \frac{\Delta}{2}\left(1 - \frac{\Phi B}{2b-1}\right)^2,$$

which is equivalent to the system:

$$\begin{cases} \dfrac{2(1-b) + \Phi(B-D)}{2b-1} & \geq \quad \Delta\left(1 - \dfrac{\Phi B}{2b-1}\right)^2 \\ \Phi(B+D) + \Delta(2b-1) & > \quad 2(1-b). \end{cases} \tag{7}$$

Full-agreement equilibrium. At a full agreement equilibrium, we have $c = 1/2$, $l = (2b-1)$ and $\lambda = 1$. Such an equilibrium exists if and only if $w_1 = \frac{\Phi d_1 + \Delta/2}{1+\Delta} \leq 1-b$. This condition can be written as $\Phi(B+D) + \Delta \leq 2(1-b)(1+\Delta)$, equivalent to:

$$\Phi(B+D) + \Delta(2b-1) \leq 2(1-b). \tag{8}$$

Equilibrium uniqueness. Let $h_1 \in (0,1)$ be the solution to

$$\frac{2(1-b) + \Phi(B-D)}{2b-1} = \Delta\left(1 - \frac{\Phi B}{2b-1}\right)^2 \tag{9}$$

and $h_2 \in (0,1)$ be the solution to

$$\Phi(B+D) + \Delta(2b-1) = 2(1-b). \tag{10}$$

Note first that h_1 and h_2 do exist, since Φ is increasing with h, with $\Phi(h=0) = 0$ and Δ is decreasing with h, with $\Delta(h=1) = 0$. Second, observe that h_1 is solution to:

$$\Phi(h)(D-B) + \Delta(h)(2b-1)\left(1 - \frac{\Phi(h_1)B}{2b-1}\right)^2 = 2(1-b). \tag{11}$$

Then, comparing equations (10) and (11), noting that Φ is increasing, $D - B \leq D + B \leq 0$, that Δ is decreasing and $0 \leq \left(1 - \frac{\Phi(h_1)B}{2b-1}\right)^2 < 1$, we obtain that $h_1 \leq h_2$ (with an equality only if $B = 0$). The end of the proof proceeds as for Proposition 4.

Probability of reaching a deal and thresholds as a function of b.

Using the formula in Proposition 8, it is immediate that λ decreases with b for a two-sided equilibrium. A one-sided equilibrium is characterized by:

$$\lambda + \Delta\lambda^2/2 = \frac{b - \Phi d_1}{2b - 1} = \frac{1 - \Phi d_1/b}{2 - 1/b}.$$

This function is decreasing with b, and thus λ decreases with b for a one-sided equilibrium.

By equation (10), h_2 is defined as the smallest value of h for which $g_2(b, h)$ is below 2, with $g_2(b, h) = \Phi(h)(B + D) - \Delta(h) + 2b(\Delta(h) + 1)$. This function is increasing with b. Hence, h_2 must be increasing with b.

By equation (9), h_1 is defined as the smallest value of h for which $g_1(b, h)$ is below 2, with

$$g_1(b, h) = \Phi(h)(D - B) + (2b - 1)\Delta(h)\left(1 - \frac{\Phi(h)B}{2b - 1}\right)^2 + 2b.$$

This function is increasing with b. Hence, h_1 must be increasing with b. ■

A.9 Proof of Theorem 3

Proof. The proof closely follows that of Theorem 2. For any $\theta \in \{0, 1\}$, we have:

$$\begin{aligned} W_\theta = \frac{1}{\beta(1-h)}\left(w_\theta - hd_\theta\right) &= \frac{1}{\beta(1-h)}\left(1 - b + \left(\frac{2b-1}{2}\right)(1 - \lambda) - \frac{hD}{2}\right) \\ &= \frac{1}{2\beta(1-h)}\left(1 - (2b-1)\lambda - hD\right). \end{aligned}$$

Then, we may write:

$$\begin{aligned}
&\frac{1-\Phi D}{2b-1} = \lambda + \Delta\lambda^2 \\
&\quad \Leftrightarrow \quad 1 - \frac{h}{1-\beta(1-h)} D = (2b-1)\left(\lambda + \frac{\beta(1-h)}{1-\beta(1-h)}\lambda^2\right) \\
&\quad \Leftrightarrow \quad 1 - \beta(1-h) - hD = (2b-1)\left(\left(1-\beta(1-h)\right)\lambda + \beta(1-h)\lambda^2\right) \\
&\quad \Leftrightarrow \quad 1 - (2b-1)\lambda - hD = \beta(1-h)\left(1 - (2b-1)\lambda + (2b-1)\lambda^2\right).
\end{aligned}$$

We thus obtain: $W_0 = W_1 = \frac{1}{2} + \frac{2b-1}{2}(-\lambda + \lambda^2)$, as desired. ■

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