The Marginal Voter's Curse*

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This Version: May 16, 2017
First Version: October 1, 2014.

Abstract

Recent empirical evidence suggests that policy outcomes are responsive to electoral margins, even away from the 50% threshold. If the impact of a vote is frequent and small, rather than rare and huge, however, then the swing voter's curse, useful for explaining patterns of voter participation, should not arise. Nevertheless, we show in this paper that this opposite voting calculus generates a new reason for abstention, to avoid the *marginal voter's curse* of diluting the pool of information. Surprisingly, the marginal voter's curse turns out to be stronger than the swing voter's curse. In fact, in a model with both incentives, marginal considerations come to completely dominate pivotal considerations as the electorate grows large, revealing that predictions based solely on the standard voting calculus are knife-edge.

JEL classification: C72, D70

Keywords: Turnout, Information aggregation, Underdog effect

*We thank participants at the Political Economy Workshops at Alghero, Bath, Lancaster University, Mont Tremblant and at the Wallis Institute. We also thank seminar participants at Brigham Young University, Caltech, Carlos III, CERGE-EI, European University Institute, NYU Abu Dhabi, Queen Mary University of London, Simon Frazer University, UC Berkeley, UCL, UC San Diego, Università di Bologna, Université de Montréal, University of British Columbia, University of Hawaii, University of Mannheim, University of Portsmouth, University of Queensland, University of Surrey, University of Tokyo, University of Toronto, University of Warwick, University of Western Ontario. We particularly thank Dan Bernhardt, Chris Bidner, Laurent Bouton, Alessandra Casella, Micael Castanheira, Jon Eguia, Tim Feddersen, Faruk Gul, Wei Li, Claudio Mezzetti, David Myatt, Santiago Oliveros, Louis Philippos, Carlo Prato, and Francesco Trebbi for helpful comments and suggestions. We would also like to thank Bruno Nogueira Lanzer for excellent assistance.

1 Introduction

In existing models of large majoritarian elections, a central tenet is that voters restrict attention to the rare occasion in which a vote is pivotal, meaning that it changes the identity of the election winner. Mechanically, of course, this is indeed the only function a vote is meant to perform. That voters seem at best vaguely aware of pivotal voting considerations, however, has prompted a conjecture that voters might perceive (rightly or wrongly) their votes to have broader impact than this.[1] Perhaps, for example, adding to the winning candidate's margin of victory emboldens him or her to pursue more extreme policies.[2] Empirically, it does appear that candidates who win by larger margins are more extreme.[3] If so, however, then since every vote contributes (positively or negatively) to the margin of victory, every vote should have a marginal impact on the policy outcome, and rational voters should take this into account; in that case, the standard pivotal voting calculus is wrong—or at least incomplete.[4]

One setting where this restriction has an important impact is voter participation, a political behavior that has generated perhaps more discussion than any other. Such interest arises not only because variations in voter participation influence electoral results, political positions, and ultimately policy outcomes, but also because turnout is one of the most visible decisions that voters make, thus offering a possible window into voter motivations and rationality. For example, a seminal paper by Feddersen and Pesendorfer (1996) proposes an *information* model of voter participation, where citizens vote if they are confident in their understanding of the issues and candidates, but otherwise strategically abstain, delegating the decision to those who know more, to avoid the *swing voter's curse* of overturning an informed decision. Such behavior

[1]As Austen-Smith and Banks (1996) show, the pivotal voting calculus can also generate rather pathological behavior, such as voting for the candidate that seems inferior.

[2]This captures the popular notion of electoral *mandates*, which are variously formalized by Castanheira (2003), Razin (2003), Shotts (2006), Fowler and Smirnov (2007), and McMurray (2017).

[3]Conley (2001) documents this for the US presidency, Fowler (2005) for the US Senate, and Faravelli, Man and Walsh (2015) for the US House of Representatives. See also Fowler (2006) and Bernhard et al. (2008).

[4]Blais (2000) argues that voters seem only vaguely aware of pivot probabilities; conceivably, a marginal impact on policy might correspond to a voter's perception of the pivotal impact of a vote.

seems natural, and is consistent with a large literature identifying information as the key empirical determinant of voter participation.[5] Unlike earlier literature that attributes abstention to voting costs, it also provides a plausible rationale for why citizens might abstain when voting is costless, for example skipping races on a ballot after voting costs are sunk.[6] That one voter is willing to rely on others' opinions of what is to be done suggests that voters ultimately share a common interest; thus, another legacy of that paper is the resurgence of the classic *common value* paradigm of Condorcet (1785), where elections serve to pool information rather than resolve conflicts of interest.[7] In the end, however, the swing voter's curse stems from the pivotal voting calculus; if this calculus is not relevant then the swing voter's curse cannot arise.[8]

This paper proposes a common value model of voter turnout that takes both the pivotal and the marginal impact of a vote into account. We include pivotal voting incentives by allowing the policy outcome to jump discontinuously when one party's vote share crosses the 50% threshold, but importantly, we also allow votes to have a marginal impact on the policy outcome away from this threshold, as depicted in Figure 1. The main result is that citizens with low (though still positive) levels of information abstain, even when voting is costless. As long as a pivot remains at the 50% threshold, this is not surprising; however, we show that abstention occurs even when this pivot is removed entirely. This is for a new reason, which we call the

[5] For reviews of this extensive empirical literature, see Triossi (2013) and McMurray (2015). In particular, turnout and roll-off are both correlated with political knowledge and with other variables associated with information, such as education and age. Evidence from the natural experiment of Lassen (2005) and the field experiments of Banerjee et al. (2011) and Hogh and Larsen (2016) suggest that improving information has a causal impact on voter participation.

[6] Wattenberg, McAllister, and Salvanto (2000) find information to be the most important determinant of casting complete ballots.

[7] For example, see Feddersen and Pesendorfer (1997, 1998, 1999), Piketty (1999), Myerson (2002), Razin (2003), Martinelli (2006), Krishna and Morgan (2011), Ahn and Oliveros (2012, 2016), Bouton and Castanheira (2012), Bhattacharya (2013), McMurray (2013, 2017a,b,c,d), Bouton, Llorente-Saguer, and Malherbe (2016), Ekmekci and Lauermann (2016), Osborne, Rosenthal, and Stewart (2016), Ali, Mihm, and Siga (2017), Barelli, Bhattacharya, and Siga (2017), and Battaglini (2017).

[8] The pivotal voting logic that produces the swing voter's curse is that, when others are voting informatively, the party with the superior policy position is more likely to lead by one vote than to trail by one vote, so an individual is more likely to be pivotal when he mistakenly votes for the party with the inferior policy position than when he correctly votes for the superior party.

marginal voter's curse of nudging the policy in the wrong direction. Rather than hinging on the huge impact that a vote has under rare circumstances, the marginal voter's curse stems from the tiny impact that a vote has in universal circumstances. In that sense, the marginal voter's curse and the swing voter's curse are exact opposites.

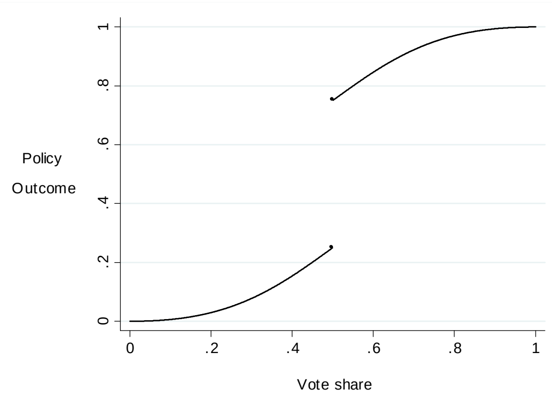

Figure 1: Mapping between vote shares and policy outcomes.

The logic of the marginal voter's curse is that a vote for the losing party has a greater impact on the winning party's vote share than an additional vote for the winning party has. If a voter trusts his peers to vote informatively, he expects the party with the superior policy position to be ahead. Thus, if he is uncertain which party is superior, he should abstain from voting, because the benefit that his vote will generate if his private opinions are correct is smaller than the damage his vote will inflict if he is in error. While the marginal voter's curse and swing voter's curse arise for opposite reasons, they generate similar comparative statics with regard to the underlying distribution of voter preferences and information, and can be viewed as two manifestations of a seemingly unrelated general phenomenon, the *underdog effect* that has been noted in existing literature on voter participation.

Intuitively, it might seem that comparing nudges in one direction or the other would have less impact on voter beliefs than conditioning on such an unusual event as a pivotal vote. However, the marginal voter's curse turns out to be stronger than the swing voter's curse, in the sense that abstention is higher in a pure marginal voting model than in a pure pivotal voting model.[9] In a general model that includes

[9] Note that this prediction is opposite of that obtained in the private value setting of Herrera,

both pivotal and marginal considerations, of course, both curses operate. As the number of votes grows large, however, the importance of pivotal voting considerations shrinks relative to marginal voting considerations. In the limit, even if the marginal impact of a vote is only minimal, voter participation converges to the same level that would prevail if there were no discontinuity at all at the 50% threshold. In other words, a model with both pivotal and marginal voting considerations makes the same predictions for large elections as a model with marginal voting considerations alone. In that sense, ignoring marginal voting incentives not only fails to capture an aspect of elections that is relevant empirically; it also generates predictions that turn out to be knife-edge in a more general setting.

An intuition for why the marginal voter's curse is stronger than the swing voter's curse is as follows. When all that matters is which side receives a majority of votes, a single mistaken vote for the political party with the inferior policy platform can be entirely remedied by a single correct vote for the party with the superior policy position. The same is not true when margins matter, because vote shares become diluted, so a vote for the majority party has a lower impact on policy than a vote for the minority. As a simple illustration of this, suppose that the better of two alternatives received three out of five votes, or a 60% vote share. One additional vote for the opposite party reduces this vote share to 50% (three out of six), and an additional vote of support brings it back up, but only to 57% (four out of seven). Thus, it takes more than one vote to compensate for a mistake. In that sense, mistakes are more permanent when a vote has a marginal impact on policy than when it doesn't, and voters work harder to avoid them. In essence, it is not sufficient to give lots of votes to the superior side; the electorate must also give as few votes as possible to the inferior side, so that the better side not only wins, but wins by a large margin. This generates the underdog effect, where a vote for the majority party has a lower impact on policy than a vote for the minority.

The remainder of this paper is organized as follows. Section 2 discusses related literature, after which Section 3 introduces the formal model. The general model includes both pivotal and marginal voting incentives, but Sections 4 and 5 begin

Morelli, and Palfrey (2014), where abstention is higher in a pure pivotal voting model than in a pure marginal voting model, as long as support for the two parties is not precisely balanced.

by isolating marginal incentives and then pivotal incentives, respectively. Section 6 compares these polar cases, and then Section 7 analyzes the general model, and shows that it converges in large elections to the pure marginal voting case considered in Section 4. For simplicity, all of these sections treat a linear policy response function, but Section 8 extends the model to nonlinear policy functions such as the one pictured in Figure 1. Section 9 then concludes, and proofs of theoretical results are presented in the Appendix.

2 Literature

Since the seminal work of Palfrey and Rosenthal (1983, 1985), game-theoretic literature on voter participation has grown extensively. Most of these papers restrict attention to a private-value setting, however, and consider only pivotal motives, in contrast with the empirical evidence cited above. A more recent set of papers by Castanheira (2003), Shotts (2006), Meirowitz and Shotts (2009), Herrera, Morelli, and Palfrey (2014), Faravelli, Man, and Walsh (2015), Faravelli and Sanchez-Pages (2015), Herrera, Morelli, and Nunnari (2015) and Kartal (2015) explore marginal voting incentives in a private-value setting with costly voting.[10]

To the best of our knowledge, this is the first paper that features both pivotal and marginal motives in a common-value environment. Feddersen and Pesendorfer (1996) identify the swing voter's curse and Krishna and Morgan (2012) and McMurray (2013) extend this to more general distributions of private information. The latter formulation is used in the model below. Other papers in this literature, such as Feddersen and Pesendorfer (1999) and Krishna and Morgan (2011), focus instead on generalizing to heterogeneous preferences. In all of these papers, attention is restricted to the pivotal impact of a vote. Razin (2003) proposes a model in which voting has a marginal impact on candidates' policy beliefs, but does not consider abstention. In that case, McMurray (2017c) demonstrates a "signaling voter's curse" that leads relatively uninformed citizens to abstain, but that result relies on can-

[10] Marginal incentives have also been explored in other settings such as political competition or institutional comparisons. See, for example, Ortuño-Ortín (1997), Llavador (2006, 2008) Iaryczower and Mattozzi (2013), or Matakos, Torumpounis and Xefteris (2015a,b).

didates sharing voters' preferences, and learning from abstention just as they learn from votes. In contrast, the marginal voter's curse arises with a mapping from vote shares to policy outcomes that is purely mechanical. Together, the swing voter's curse, signaling voter's curse, and marginal voter's curse suggest that the incentive to abstain is a robust consequence of common values and heterogeneous expertise, and not an artifact of any specific political institution.

3 The Model

An electorate consists of N citizens where, following Myerson (1998), N is finite but unknown, and follows a Poisson distribution with mean n. Together, these citizens must choose a policy from an interval. There are two political parties, each with policy positions in the interval. At the beginning of the game, and with equal probability, Nature designates one of these policy positions as ultimately better for society than the other. Let A denote the party with the superior position and B denote the party with the inferior position. Letting 0 denote the inferior policy position and 1 denote the superior position, $x \in [0, 1]$ can denote any policy between the two parties' positions and also the social welfare $u(x) = x$ that will be attained if that policy is implemented.

Citizens are each independently designated as one of two types. With probability $2p$, a citizen is a partisan, and with equal probability favors one party or the other, regardless of which policy position Nature designated as superior. With remaining probability $I = 1 - 2p$, a citizen is designated as *independent* or *non-partisan*. Independents prefer to do whatever is socially optimal, evaluating policy x according to the welfare function $u(x)$ given above. From an independent's perspective, each of his fellow citizens has probability p of being a partisan supporter of the superior party A and probability p of being a partisan supporter of the inferior party B.

The policy outcome is a general function $x : \mathbb{Z}_+^2 \to [0, 1]$ of the numbers a and b of votes that are cast for either party, which can be most easily described in terms of two benchmark cases. The first is the case of pure *marginal voting*, which means that the policy outcome is a weighted average of the parties' policy positions, with weights

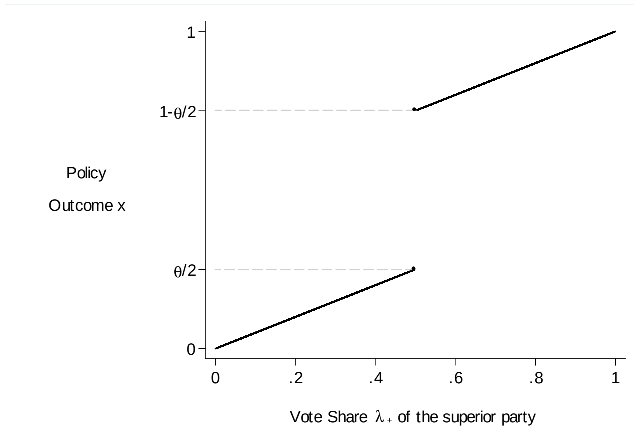

Figure 2: Mapping between vote shares and policy outcomes.

given by the parties' vote shares. That is, if a fraction $\lambda_+ = \frac{a}{a+b}$ of the electorate votes for party A and a fraction $\lambda_- = \frac{b}{a+b}$ votes for B then the policy outcome is given by $x(a,b) = 0\lambda_- + 1\lambda_+ = \lambda_+$, ranging continuously from 0 to 1 are the vote share of the superior party ranges from 0% to 100%.[11] The second benchmark is the more traditional case of pure *pivotal voting*, which corresponds to the mechanical impact of a vote. In that case, x is simply the policy position $x_w \in \{x_A, x_B\}$ of the party w who wins the election (i.e. 0 if $b > a$ and 1 if $a > b$, breaking a tie if necessary by a fair coin toss). The general model admits both types of voting incentives: the policy outcome is given by

$$x = \theta\lambda_+ + (1-\theta)\, x_w \tag{1}$$

with $\theta \in [0,1]$. This formulation is a weighted average of pure marginal voting and pure pivotal voting, which correspond to $\theta = 1$ and $\theta = 0$, respectively. As Figure 2 illustrates, policy shifts discontinuously when one party's vote share crosses the 50% threshold, consistent with the mechanical impact of a vote, but even away from this threshold, changes in one party's vote share push the policy outcome marginally in that party's direction.

The optimal policy cannot be observed directly, but independent voters observe

[11] An alternative assumption that would lead to identical analysis is that policy 1 is implemented with probability λ_+ and policy 0 is implemented with probability λ_-, and that independent voters are risk neutral. This could result from probabilistic voting across independent legislative districts, as in Levy and Razin (2015).

private signals s_i that are informative of Nature's choice.[12] These signals are of heterogeneous quality, reflecting the fact that citizens differ in their expertise on the issue at hand. Specifically, each citizen is endowed with information quality $q_i \in \mathcal{Q} = [0, 1]$, drawn independently according to a common distribution F which, for simplicity, is continuous and has full support. Conditional on $q_i = q$, a citizen's signal correctly identifies the party whose policy position is truly superior with the following probability.

$$\Pr(s_i = A|q) = \frac{1}{2}(1+q) \tag{2}$$

With complementary probability, a citizen mistakes the inferior party for the superior party.

$$\Pr(s_i = B|q) = \frac{1}{2}(1-q) \tag{3}$$

With this specification, q_i can be interpreted as the correlation coefficient between a voter's private opinion and the truth. That is, a citizen with $q_i = 1$ is perfectly informed about Nature's choice, while a signal with $q_i = 0$ is completely uninformative. An independent can vote (at no cost) for the party that he perceives to be superior, or can abstain.[13] Let $\sigma : \mathcal{Q} \to [0, 1]$ denote a (mixed) participation strategy, where $\sigma(q)$ denotes the probability of voting for an individual with expertise $q \in \mathcal{Q}$, and let Σ denote the set of such strategies. By Bayes' rule, (2) and (3) can be reinterpreted as a voter's posterior belief that he has correctly voted for the superior party or mistakenly voted for the inferior party, respectively.

Given a participation strategy, the probabilities with which a citizen votes for party A and party B, respectively, are given by the following.

$$v_+ = p + I\int_0^1 \sigma(q)\frac{1}{2}(1+q)\,dF(q) \tag{4}$$

$$v_- = p + I\int_0^1 \sigma(q)\frac{1}{2}(1-q)\,dF(q) \tag{5}$$

These include the probability p of favoring either party for partisan reasons, as well as the probabilities of voting as an independent with any level of expertise. Together,

[12] Partisans could receive signals as well, of course, but would ignore them in equilibrium.

[13] A strategy of voting against one's signal could be allowed but would not be used in equilibrium.

(4) and (5) also determine the level $v_\tau = v_+ + v_-$ of voter turnout.

If every citizen follows the same participation strategy, (4) and (5) can be interpreted as the expected vote shares of the superior and inferior parties, respectively. By the decomposition property of Poisson random variables (Myerson 1998), the numbers a and b of votes for the superior and inferior parties, respectively, are independent Poisson random variables with means $n_+ = nv_+$ and $n_- = nv_-$. Thus, the probability of exactly a votes for the superior party and b votes for the inferior party is the product

$$\Pr(a,b) = \frac{e^{-n_+} n_+^a}{a!} \frac{e^{-n_-} n_-^b}{b!} \tag{6}$$

of Poisson probabilities. Similarly, the expected total number of votes can be written as $n_\tau = nv_\tau$.

By the environmental equivalence property of Poisson games (Myerson 1998), an individual from within the game reinterprets a and b as the numbers of correct and incorrect votes cast by his peers; by voting himself, he might add one to either total. When there are a votes for the superior party and b votes for the inferior party, the change in utility $\Delta_+ x(a,b)$ from contributing one additional vote for the superior party and the change in utility $\Delta_- x(a,b)$ from accidentally adding one vote for the inferior party are given by the following.

$$\Delta_+ x(a,b) = x(a+1,b) - x(a,b) \tag{7}$$

$$\Delta_- x(a,b) = x(a,b+1) - x(a,b) \tag{8}$$

The magnitudes of these utility changes depend on the numbers of votes cast for either side by a citizen's peers; averaging over all possible voting outcomes, the expected benefit of voting is given by

$$\Delta Eu(q) = E_{a,b}\left[\frac{1}{2}(1+q)\,\Delta_+ x(a,b) + \frac{1}{2}(1-q)\,\Delta_- x(a,b)\right] \tag{9}$$

which depends on a citizen's expertise q. Implicitly, the expectation in (9) depends on the voting strategy adopted by a citizen's peers. If his peers all follow the strategy $\sigma \in \Sigma$, a citizen's best response is to vote if his q is such that (9) is positive and to abstain otherwise. A strategy σ^* that is its own best response constitutes a (symmetric) Bayesian Nash equilibrium of the game.

Sections 4 and 5 analyze incentives for equilibrium behavior under the two benchmark cases of pure marginal voting ($\theta = 1$) and pure pivotal voting ($\theta = 0$), respectively. Section 6 then compares equilibrium levels of participation for these two benchmarks. Section 7 next analyzes equilibrium behavior for the general case, along with social welfare, and extends the model to a more general relationship between vote totals and the policy outcome.

4 Marginal Voting

If $\theta = 1$ then the general model of Section 3 reduces to the benchmark case of pure marginal voting. In that case, the policy outcome $x = \lambda_+$ is simply the vote share of the party with the superior policy position. Changes in utility (7) and (8) from an additional vote for the superior party and from an additional vote for the inferior party can then be written as

$$\Delta_+ x(a,b) = \frac{a+1}{a+b+1} - \frac{a}{a+b} = \Delta\lambda_+ \tag{10}$$

$$\Delta_- x(a,b) = \frac{a}{a+b+1} - \frac{a}{a+b} = -\Delta\lambda_- \tag{11}$$

in terms of the increases $\Delta\lambda_+ = \frac{a+1}{a+b+1} - \frac{a}{a+b}$ and $\Delta\lambda_- = \frac{b+1}{a+b+1} - \frac{b}{a+b}$ in these vote shares that an additional correct vote or incorrect vote cause, respectively.

Since $\Delta\lambda_+$ and $\Delta\lambda_-$ are both positive, (9) is increasing in q, and is therefore positive for all q above some threshold $\bar{q}$. In other words, as Theorem 1 states below, the best response to any strategy σ can be characterized as a *threshold strategy* $\sigma_{\bar{q}}$, meaning that a citizen votes if his expertise exceeds a threshold $\bar{q}$, but abstains otherwise. Specifically, (9) is positive if and only if q exceeds $\bar{q}_M^{br}$, defined as follows.

$$\bar{q}_M^{br} = \frac{E_{a,b}(\Delta\lambda_-) - E_{a,b}(\Delta\lambda_+)}{E_{a,b}(\Delta\lambda_-) + E_{a,b}(\Delta\lambda_+)} \tag{12}$$

From (4) and (5) it is clear that $v_+ > v_-$ for any strategy in which a positive fraction of the electorate votes. Because of this, the number a of votes for the party with the superior policy position is likely to exceed the number b of votes for the opposing party. This is good for welfare, but since vote shares respond more strongly to an additional vote for the party that is behind than to an additional vote for the party that is ahead, as explained in Section 1, it also implies that $E_{a,b}(\Delta\lambda_-) > E_{a,b}(\Delta\lambda_+)$, and therefore that $\bar{q}_M^{br} > 0$.[14] In other words, the best response for citizens with the lowest levels of expertise is to abstain from voting—even though voting is costless and the swing voter's curse is not relevant—to avoid the *marginal voter's curse* of pushing the policy outcome in the wrong direction. Equilibrium existence follows from standard fixed point arguments.

Theorem 1 (Marginal Voter's Curse) *If $\theta = 1$ then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a threshold strategy $\sigma_{\bar{q}_M^*}$ with $\bar{q}_M^* > 0$. Moreover, such an equilibrium exists.*

The result that independent voters each receive informative private signals but not all report their signals in equilibrium implies that valuable information is lost. Intuitively, this may seem to justify efforts to increase voter participation, for example by punishing non-voters with stigma or fines. To the contrary, however, McLennan (1998) shows that, in common-value environments such as this, whatever is socially optimal is also individually optimal, implying that equilibrium abstention in this setting actually improves welfare. To see how it can be welfare improving to throw away signals, note that citizens actually have not one but two pieces of private information: their signal realization s_i and their expertise q_i. In an ideal electoral system, all signals would be utilized, but would be weighted according to their underlying expertise. Here, votes that are cast are instead weighted equally. Abstention provides a crude mechanism whereby citizens can transfer weight from the lowest quality signals to those that reflect better expertise.

Theorem 1 applies for a population of any size n. As actual electorates tend to be extremely large, however, the rest of this section now derives the limit of equilibrium behavior as n grows large. Such asymptotic results are made possible by Lemma 1,

[14] Section 8 uses this *underdog property* to generalize the marginal voting model to allow nonlinear functions of the vote totals.

which offers an algebraic simplification of the formulas obtained previously, in terms of the expected fractions v_+ and v_- of the electorate who vote for the parties with the superior and inferior policy positions, respectively, and the total fraction v_τ who turn out to vote.

Lemma 1 *The following hold for any* n *and for any threshold strategy* $\sigma_{\bar{q}}$*.*

$$E_{a,b}(\Delta\lambda_+) = \frac{v_-}{nv_\tau^2} + \frac{n\left(v_+^2 - v_-^2\right) - 2v_-}{2nv_\tau^2} e^{-nv_\tau} \tag{13}$$

$$E_{a,b}(\Delta\lambda_-) = \frac{v_+}{nv_\tau^2} + \frac{n\left(v_-^2 - v_+^2\right) - 2v_+}{2nv_\tau^2} e^{-nv_\tau} \tag{14}$$

With Lemma 1, an equivalent condition to equation (12) is that $\bar{q}_M^{br}$ solves the following.

$$\frac{1+\bar{q}}{1-\bar{q}} = \frac{2v_+ + \left[n\left(v_+^2 - v_-^2\right) - 2v_-\right] e^{-nv_\tau}}{2v_- + \left[n\left(v_-^2 - v_+^2\right) - 2v_+\right] e^{-nv_\tau}} \tag{15}$$

As n grows large, the right-hand side of this expression converges simply to the likelihood ratio $\rho(\bar{q}) = \frac{v_+}{v_-}$ of correct to incorrect votes. Since (15) is continuous both in $\bar{q}$ and in n, therefore, the limit $q^M = \lim_{n\to\infty} \bar{q}_n^*$ of any sequence of equilibrium threshold must solve the following simpler equation,

$$\rho(\bar{q}) = \frac{1+\bar{q}}{1-\bar{q}} \tag{16}$$

which can also be written in terms of the vote share λ_+:[15]

$$\lambda_+ = \frac{1}{2}(1+\bar{q}) \tag{17}$$

[15] The convergence of (15) to (16) is not trivial—because v_+, v_-, and v_τ change with $\bar{q}_n^*$, which changes with the population size—but nevertheless holds, as the proof of Proposition 1 demonstrates formally.

For a threshold strategy, v_+ and v_- reduce from (4) and (5) to the following,

$$\begin{aligned} v_+ &= p + I \int_{\bar{q}}^{1} \frac{1}{2}(1+q)\, dF(q) \\ &= p + \frac{1}{2} I \left[1 - F(\bar{q})\right] \left[1 + m(\bar{q})\right] \end{aligned} \tag{18}$$

$$\begin{aligned} v_- &= p + I \int_{\bar{q}}^{1} \frac{1}{2}(1-q)\, dF(q) \\ &= p + \frac{1}{2} I \left[1 - F(\bar{q})\right] \left[1 - m(\bar{q})\right] \\ v_\tau &= 2p + I \left[1 - F(\bar{q})\right] \end{aligned} \tag{19}$$

where $m(\bar{q}) = E(q | q > \bar{q})$ denotes the average expertise among citizens who actually vote, and the left-hand side of (16) can therefore be rewritten as

$$\rho(\bar{q}) = \frac{K + \left[1 - F(\bar{q})\right]\left[1 + m(\bar{q})\right]}{K + \left[1 - F(\bar{q})\right]\left[1 - m(\bar{q})\right]} \tag{20}$$

in terms of the ratio $K = \frac{2p}{I}$ of partisans to independents.

The proof of Proposition 1 shows that equation (16) has a unique solution q^M. Uniqueness in the limit does not imply a unique equilibrium in any game with finite size parameter n. But if there are multiple equilibrium participation thresholds then the implication of Proposition 1 is that these all converge to each other in the limit. A unique limiting participation threshold of course translates into a unique limiting level $v_\tau = v_+ + v_-$ of expected voter participation, and actual turnout in large elections converges to its expectation. The margin of victory $\mu = \frac{v_+ - v_-}{v_\tau}$ in a large election is determined by the same threshold.

In addition to stating uniqueness in the limit, Proposition 1 derives some comparative static implications of the limiting equilibrium condition (16). Intuitively, it might seem that the marginal voter's curse should attenuate as n grows large, because the damage caused by one mistaken vote shrinks, so citizens should be less worried about making mistakes. If so, abstention should decline as the electorate grows large, and turnout should tend toward 100% in the limit. Contrary to this intuition, however, the first part of Proposition 1 states that q^M is strictly positive, meaning that a positive fraction of the electorate continue to abstain no matter how large the electorate grows. In fact, if there are no partisans then q^M equals one, meaning that—in sharp contrast with the intuition above—turnout tends to 0% in the limit.

Proposition 1 *There exists a unique* q^M *such that, for any sequence* $\bar{q}_M^*(n)$ *of equilibrium thresholds under pure marginal voting,* $\lim_{n\to\infty} \bar{q}_M^*(n) = q^M$. *Moreover,* q^M *exhibits the following properties:*

(i) $0 < q^M < 1$ *for any partisan share* $p > 0$. *If* $p = 0$ *then* $q^M = 1$.

(ii) q^M *decreases strictly with* p.

(iii) If $p > 0$ *then improvements in the distribution* F *of expertise that satisfy the monotone likelihood ratio property increase* q^M.

Intuitively, the reason why the incentive to abstain does not vanish in large elections is that the policy outcome is a weighted average of the two extremes, with weights corresponding to vote shares. Citizens wish to vote as unanimously as possible in favor of the superior side, and this is accomplished by limiting participation to those who are the least likely to err. That $q^M = 1$ when there are no partisans is a consequence of the assumption that the distribution F of expertise has full support; more generally, if the maximum level of expertise is $q_{\max}$ then, when there are no partisans, $q^M = q_{\max}$. Either way, the entire electorate defers to the vanishing segment of the electorate who are least likely to dilute the electoral outcome with incorrect votes.

The limiting level of equilibrium participation can be most easily compared with that of pure pivotal voting by solving the limiting equilibrium condition (16) for K, as follows.

$$\frac{1 - F(\bar{q})}{\bar{q}} \left[m(\bar{q}) - \bar{q} \right] = K \tag{21}$$

This also facilitates numerical examples, as the abstention rate $F(\bar{q})$ and conditional mean $m(\bar{q})$ can easily be computed for specific distributions of expertise. The simplest example of this is a uniform distribution, for which $F(\bar{q}) = \bar{q}$ and $m(\bar{q}) = \frac{1+\bar{q}}{2}$,

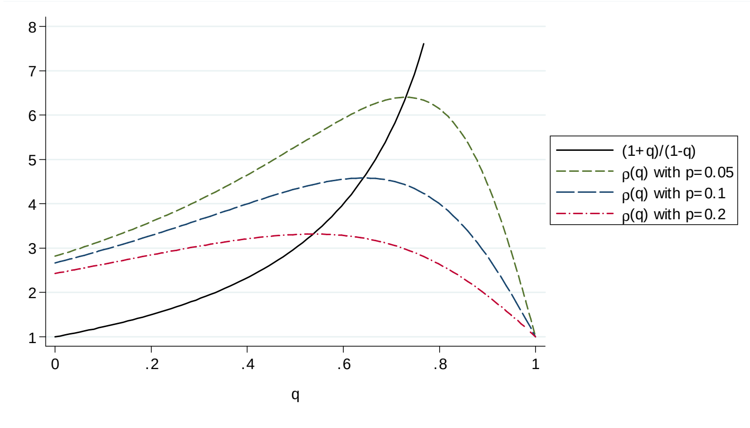

Figure 3: Left and right-hand side of equation (16) when the distribution F of expertise is uniform for various levels of partisans.

so that (21) reduces to

$$q^M = (K+1) - \sqrt{K(K+2)}. \tag{22}$$

Using the uniform distribution, Figure 3 plots the left- and right-hand sides of (16) for various levels p of partisanship.

Evidently, the left-hand side of (16) is maximized precisely at the intersection of the two. Indeed, in demonstrating uniqueness, the proof of Proposition 1 shows that this must always be the case. The intuition for Proposition 1 is related to the intuition for this phenomenon. To see this, first note that $\rho(\bar{q})$ represents the likelihood ratio of a correct vote to an incorrect vote, for a randomly chosen voter. The objective of independent voters is precisely to make this ratio as large as possible, so that the policy outcome will be as close as possible to whatever is truly optimal. The right-hand side of (16) is the likelihood ratio of a correct vote to an incorrect vote for the marginal independent voter—that is, one whose expertise is right at the participation threshold. Equilibrium equates the average and the margin.[16] Equating the average and marginal likelihood ratios serves to maximize the average, just as equating the average and marginal costs of a firm's production serves to

[16]Equivalently, as is clear from (17), equilibrium equates the vote share of the superior party with the posterior beliefs of the marginal voter.

minimize the average: if the marginal voter's likelihood ratio is not as good as the average voter's, increasing the participation threshold removes votes of below-average quality, thus improving the average; if the marginal voter's likelihood ratio is better than the average voter's, raising the participation threshold removes votes of above-average quality, thus making things worse.

The second part of Proposition 1 states that the marginal voter's curse is most severe when there are fewer partisans. Since partisans always vote, this implies that turnout is lower with fewer partisans, as well. With no partisans at all, average quality always exceeds marginal quality, because the marginal voter is precisely the one with the lowest expertise. Thus, the equilibrium threshold rises all the way to $q^M = 1$. From an independent's perspective, however, adding partisans adds noise to the electoral process. For any participation threshold $\bar{q}$, therefore, a higher level of partisanship reduces the average accuracy of a vote, as Figure 3 makes clear. The accuracy of the marginal voter is unchanged, however, and strictly improves with $\bar{q}$, implying that the solution q^M to (16) is lower, as stated in the second part of Proposition 1. Since partisans themselves also vote, this has the clear effect of raising turnout.

The last part of Proposition 1 states that improving the distribution of expertise has the effect of raising the limiting participation threshold q^M. The intuition for this merely complements that of increasing partisanship: improving expertise improves the correct-to-incorrect vote ratio for any participation threshold $\bar{q}$, so the solution to (16) is higher than before. In the case of partisanship, lowering q^M unambiguously raises voter participation. Holding fixed the distribution of expertise, increases in q^M correspond to decreases in voter turnout. If changes in q^M are the result of changes in the distribution of expertise, however, then the effect of these changes on participation are ambiguous: on one hand, raising citizens above the participation threshold increases turnout by transforming non-voters into voters, but on the other hand, raising the participation threshold lowers turnout, by transforming voters into non-voters.

5 Pivotal Voting

If $\theta = 0$ then the general model of Section 3 reduces to the benchmark case of pure pivotal voting, consistent with the purely mechanical impact of a vote.[17] In that case, the policy outcome is a random variable x_w that equals 0 if $a < b$, 1 if $a > b$, and 0 or 1 with equal probability if $a = b$. A single vote for the superior party increases its probability of winning by the following amount,

$$\Pr(\mathcal{P}_+) = \frac{1}{2}\Pr(a = b) + \frac{1}{2}\Pr(a = b + 1) \tag{23}$$

which is the standard probability of being *pivotal* (event $\mathcal{P}_+$). Similarly, the probability with which an incorrect vote is pivotal (event $\mathcal{P}_-$) is given by the following.

$$\Pr(\mathcal{P}_-) = \frac{1}{2}\Pr(a = b) + \frac{1}{2}\Pr(b = a + 1) \tag{24}$$

A pivotal vote for the party with the superior policy position increases utility from zero to one (a change of 1) and a pivotal vote for the inferior party decreases utility from one to zero (a change of -1). Outside of these pivotal events, a vote does not change the policy outcome, and so does not impact utility; accordingly, the expected benefit (9) of voting reduces to the following.

$$\Delta Eu(q) = \frac{1}{2}(1 + q)\Pr(\mathcal{P}_+) - \frac{1}{2}(1 - q)\Pr(\mathcal{P}_-) \tag{25}$$

Since pivot probabilities are positive, (25) increases in q, implying that the best response to any strategy is again for a citizen to vote if his expertise is sufficiently high but abstain otherwise. In other words, as Theorem 2 states below, best-response voting can be characterized as a threshold strategy, just as in the case of pure marginal voting. Specifically, the best-response threshold $\bar{q}_P^{br}$ is given by the following.

$$\bar{q}_P^{br} = \frac{\Pr(\mathcal{P}_-) - \Pr(\mathcal{P}_+)}{\Pr(\mathcal{P}_-) + \Pr(\mathcal{P}_+)} \tag{26}$$

[17] The results of this section generalize those of McMurray (2013) to allow for positive partisan share p.

As noted above, $v_+ > v_-$ for any strategy in which a positive fraction of the electorate votes. Because of this, the number a of votes for the party with the superior policy position is likely to exceed the number b of votes for the opposing party. This is again good for welfare, since the party with the superior policy position is more likely to win the election by one vote than to lose by one vote, and an additional vote for the inferior party is more likely to change the election outcome than an additional vote for the superior party. In other words $\Pr(\mathcal{P}_-) > \Pr(\mathcal{P}_+)$, implying that $\bar{q}_P^{br} > 0$. For citizens with the lowest levels of expertise, then, the best response is to abstain from voting as in Feddersen and Pesendorfer (1996), to avoid the *swing voter's curse* of overturning an informed electoral decision. Thus, pure marginal voting and pure pivotal voting elicit the same type of voting behavior from citizens. As before, equilibrium existence follows from standard fixed point arguments. Also, the logic of McLennan (1998) again implies that equilibrium voter abstention improves social welfare.

Theorem 2 (Swing voter's curse) *If $\theta = 0$ then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a threshold strategy $\sigma_{\bar{q}_P^*}$ with $\bar{q}_P^* > 0$. Moreover, such an equilibrium exists.*

Next, we consider how the equilibrium threshold changes as n grows large for the case of pure pivotal voting (i.e., $\theta = 0$). Myerson (2000) provides a useful preliminary result, which is that pivot probabilities in large elections can be written as follows, where $h_1(n)$ and $h_2(n)$ both approach one as n grows large.

$$\Pr(\mathcal{P}_+) = \frac{e^{-n\left(\sqrt{v_+}-\sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}} \frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_+}} h_1(n) \tag{27}$$

$$\Pr(\mathcal{P}_-) = \frac{e^{-n\left(\sqrt{v_+}-\sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+ v_-}}} \frac{\sqrt{v_+}+\sqrt{v_-}}{\sqrt{v_-}} h_2(n) \tag{28}$$

Using these formulas, the equilibrium condition (26) converges to the following,

$$\bar{q} = \frac{\sqrt{v_+} - \sqrt{v_-}}{\sqrt{v_+} + \sqrt{v_-}}$$

which is equivalent to the following.

$$\rho(\bar{q}) = \frac{v_+}{v_-} = \left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2 \tag{29}$$

The limit q^P of any sequence $\bar{q}_P^*(n)$ of equilibrium thresholds must be a solution to this equation.[18] As before, this can be rewritten using (20), and solved for K, as follows, which makes it easy to compute q^P for specific example distributions.

$$\frac{1-F(\bar{q})}{\bar{q}}\left(\frac{(1+\bar{q}^2)}{2}m(\bar{q})-\bar{q}\right) = K. \tag{30}$$

Proposition 2 now states that a solution to (29) exists and, if the distribution F of expertise is well-behaved, this solution is unique.[19] As in the case of pure marginal voting, uniqueness in the limit implies that if multiple equilibria exist then they all converge to the same behavior, determined by the partisan share p and the distribution F of expertise. As before, these uniquely determine expected voter turnout and the expected margin of victory, as well, and in large elections actual turnout and margins of error converge to their expectations. Uniqueness also facilitates the derivation of comparative statics, which according to Proposition 2 match those of pure marginal voting: a higher partisan share leads to a lower q^P (i.e., higher participation) and a better-informed electorate leads to a higher q^P (i.e., lower participation).

Proposition 2 *If f is log-concave then there exists a unique q^P such that, for any sequence $\{\bar{q}_n^*\}$ of equilibrium thresholds under pure pivotal voting, $\lim_{n\to\infty} \bar{q}_n^* = q^P$. Moreover, q^P exhibits the following properties:*

(i) $0 < q^P < 1$

[18] This approximation actually requires that the number of votes tend to infinity, not just the number of citizens, but this is guaranteed by the result below that $q^M < 1$ no matter what fraction of the electorate is partisan.

[19] Bagnoli and Bergstrom (2005) show that log-concavity is satisfied by many of the most standard density functions, but log-concavity is actually stronger than necessary for uniqueness: one can easily construct examples that exhibit unique equilibria but are not log-concave. The important thing, as the proof of Proposition 2 indicates, is that raising the participation threshold $\bar{q}$ should not increase the average expertise $m(\bar{q})$ of citizens above the threshold too quickly. This will be the case as long as the distribution of expertise does not include atoms, or "spikes" of probability (see also McMurray 2013).

(ii) q^P *strictly decreases in* p

(ii) Improvements in the distribution F *of expertise that satisfy the monotone likelihood ratio property increase* q^P*.*

In stating that q^P is strictly positive, the first part of Proposition 2 implies that, for any level of partisanship, a positive fraction of the electorate abstain from voting, no matter how large the electorate grows. This was also true of pure marginal voting, as stated in Proposition 1. The result that q^P is also strictly less than one implies that a positive fraction of the electorate continues to vote, no matter how large the electorate grows. Under pure marginal voting, Proposition 1 states the same result for any $p > 0$ but if there are no partisans then everyone abstains in the limit but the infinitesimal fraction who are perfectly informed. Section 6 further emphasizes this difference between electoral rules.

The logic for the result that q^P decreases in p is analogous to the corresponding result for q^M: when the fraction of partisans is low, an uninformed independent worries that he will cancel the vote of a better-informed independent, but when the fraction of partisans is high, it is more likely that he is canceling the vote of a partisan; in the former case he wishes to abstain, but in the latter case he wishes to vote. Mathematically, an increase in p lowers the average vote quality for any participation threshold, and therefore the correct-to-incorrect vote ratio $\rho(\bar{q})$, which is the left-hand side of (29). Since this ratio increases in $\bar{q}$, this implies a solution that is lower than before. Similar logic underlies the last part of Proposition 2, because improving the distribution of expertise raises $\rho(\bar{q})$ for any $\bar{q}$, so the solution to (29) is higher than before.

6 Comparison

Despite their distinct logic, sections 4 and 5 emphasize the similarities between the comparative static implications of the marginal voter's curse for pure marginal voting and the swing voter's curse for pure pivotal voting. Maintaining a focus on large electorates, this section now compares the levels of equilibrium voter participation for the two benchmarks (assuming a log-concave density of expertise, so that equilibrium behavior under either electoral system is unique). Such a comparison is surprisingly unambiguous, because of the strong similarity between the limiting equilibrium conditions (16) and (29) for pure marginal and pure pivotal voting.

Intuitively, it might seem that conditioning on the event a pivotal vote should have a much greater impact on behavior than conditioning on the marginal impact of a nudge in one direction or the other—especially in large elections, where a pivotal vote is such a special event, and where the magnitude of the nudge is vanishingly small. If so, abstention should be much higher—and turnout much lower—under pure pivotal voting than under pure marginal voting. As Theorem 3 now states, however, the opposite is true: q^M exceeds q^P, meaning that voter participation is highest under pure pivotal voting.

Theorem 3 *If f is log-concave then $q^M > q^P$.*

In stating that $q^M > q^P$, Theorem 3 leaves open the possibility that the two thresholds are quite close to one another, so that the difference is negligible. For specific distributions, this is straightforward to investigate. Suppose, for example, that F is uniform and that partisans comprise one third of the electorate (i.e. $p = \frac{1}{6}$, and therefore $K = \frac{1}{2}$). From (22) and (30), this implies that $q^M \approx 0.38$ and $q^P \approx 0.19$. If votes only had a marginal impact, therefore, 62% of independents would vote in large elections; if votes only had a pivotal impact, turnout among independents in large elections would be 81%. Similar computations can be made for any level of partisanship, and corresponding turnout levels are displayed in Figure 4. Evidently, there is a substantial gap between q^M and q^P for all but the highest levels of partisanship.

The difference in turnout for the two benchmark cases is most notable when there are no partisans. In that case, as Section 4 explains, turnout under pure marginal voting tends toward 0%, because of strategic unraveling: citizens with below-average expertise abstain, so as to not bring down the average vote quality, but then the average among those who are still voting is higher, and citizens below this average abstain, and so on. Since the marginal citizen is always below average, this unraveling continues until only the infinitesimal fraction of the most expert citizens remain.

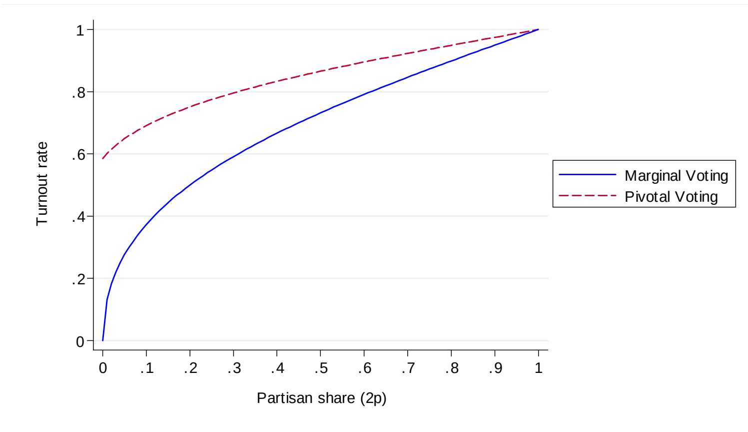

Figure 4: Turnout among independent voters as a function of the partisan share $(2p)$ when the distribution F of expertise is uniform.

Intuitively, it might seem that turnout should unravel under pure pivotal voting, as well: the marginal citizen always has less expertise than the average citizen, and so should eventually abstain, to get out of the way. The implication of Proposition 2, however, is that this does not occur: a substantial fraction of the electorate continue to vote, no matter how large the electorate grows. As McMurray (2013) explains, this reflects a trade-off between the quantity of information and the quality of information: holding the number of voters fixed, electoral outcomes are better when the expertise behind those votes is higher, which would lead people to vote, but holding expertise fixed, increasing the number of votes also improves election accuracy, just as in the classic "jury theorem" of Condorcet (1785), which gives citizens a motivation for participation. For a citizen with below-average expertise, voting decreases the average quality of information but increases the quantity. These competing considerations balance in the limit so that turnout remains substantial.

In pure marginal voting, voters face a similar trade-off, but quality is relatively more important. An intuition for why this is the case is that, when all that matters is which side receives a majority of votes, a single mistaken vote for the political party with an inferior policy platform can be offset by a single correct vote for the party with the superior policy position. The same is not true of a pure marginal voting model, because vote shares become diluted. Since a vote for the majority party has

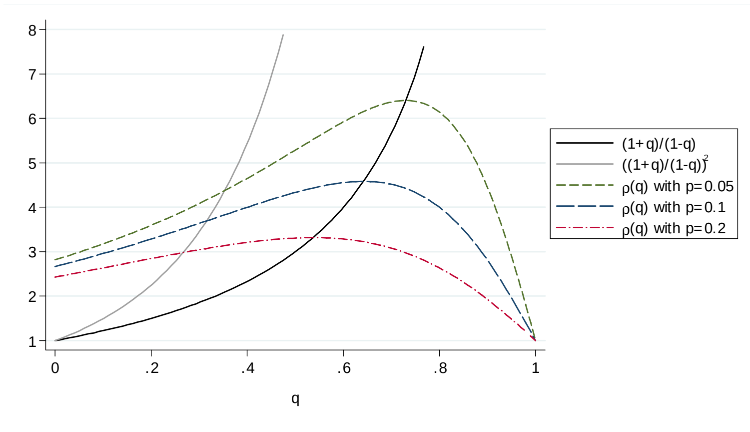

Figure 5: Left and right-hand sides of equations (16) and (29) when the distribution F of expertise is uniform for various levels of partisanship.

a lower impact on policy than a vote for the minority, it takes more than one vote to compensate for a mistake.

An alternative intuition for the discrepancy between turnout levels under pure pivotal and pure marginal voting makes use of the optimality arguments above. As Section 4 notes, the limiting equilibrium condition (16) coincides with the first-order condition for maximizing $\rho(\bar{q}) = \frac{v_+}{v_-}$—or equivalently, for maximizing $\frac{v_+}{v_+ + v_-}$. The latter is essentially an expected vote share, but in large elections this also specifies independent voters' utility, since actual vote shares converge to their expectations. In other words, $\rho(\bar{q})$ can be viewed as a monotonic transformation of voters' objective function (in large elections), and the threshold adjusts in equilibrium to the level q^M that maximizes this objective. The condition (29) for pure pivotal voting equates $\rho(\bar{q})$ to $\left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2$ instead of to $\frac{1+\bar{q}}{1-\bar{q}}$. Since the latter maximizes $\rho(\bar{q})$, the former does not, as Figure 5 illustrates for a uniform distribution. The figure also illustrates how $\left(\frac{1+\bar{q}}{1-\bar{q}}\right)^2 > \frac{1+\bar{q}}{1-\bar{q}}$ guarantees that $q^P < q^M$, which is the crux of the proof of Theorem 3.

That q^M maximizes the objective function for pure marginal voting but q^P does not begs the question of whether q^P maximizes the objective function for pure pivotal voting. Indeed, this turns out to be the case—a feature that seems not to have been noted in existing literature on majority rule. As Myerson (2002) shows, the probability with which a Poisson variable with mean nv_- exceeds an independent Poisson variable with mean nv_+ is of order $e^{-n\left(\sqrt{v_+}-\sqrt{v_-}\right)^2}$, where the exponent $\left(\sqrt{nv_+}-\sqrt{nv_-}\right)^2$ is defined in Myerson (2000) as the *magnitude* of the event.[20] The first-order condition for minimizing this magnitude is none other than the limiting equilibrium condition (29). Thus, just as q^M maximizes utility under pure marginal voting, q^P maximizes utility under pure pivotal voting. Intuitively, what matters is not only that the expectation of a exceeds the expectation of b, but also that the variances of a and b are small relative to their expectations, so that accidents in which $b > a$ do not occur.[21] The standard deviation of a Poisson random variable is the square root of its mean, so $\sqrt{nv_+}$ and $\sqrt{nv_-}$ represent the expected numbers of correct and incorrect votes, measured in standard deviations instead of in numbers of votes, and the difference between these expected vote shares is what majority voting would maximize if its impact were purely mechanical.

The results that equilibrium participation thresholds for both pure marginal and pure pivotal voting maximize their respective objective functions, but generate different levels of voter participation, begs the question of which level of participation is better for social welfare. In other words, do non-mechanical policy reactions make election outcomes better or worse? To answer this, first let x_n^P and x_n^M denote any equilibrium policy outcomes under pure pivotal and pure marginal voting, respectively, for a population size parameter n, and let $u^P = \lim_{n\to\infty} E\left(x_n^P\right)$ and $u^M = \lim_{n\to\infty} E\left(x_n^M\right)$ denote the limits of expected utility for either benchmark. u^P and u^M are well-defined because, as shown above, equilibrium behavior converges to a unique limit in either benchmark. The result that equilibrium abstention improves welfare, together with the result that abstention is higher in the pure marginal voting benchmark, might seem to suggest that welfare is higher for that case than for the pivotal voting benchmark. As Proposition 3 now states, however, the opposite is true.

Proposition 3 $u^P = 1$ *for all* $p < \frac{1}{2}$. u^M *decreases in* p, *with* $u^M = 1$ *when* $p = 0$ *and* $u^M = \frac{1}{2}$ *when* $p = \frac{1}{2}$.

[20] In addition to measuring the magnitude of the event of a win, this quantity measures the magnitude of the event of a tie, which is the smallest deviation from the expected outcome.

[21] These two considerations correspond to the considerations of quality and quantity, discussed above.

The comparison here of welfare has little to do with the turnout comparison from Theorem 3. What drives the result is that, under pure pivotal voting, A partisans and B partisans negate one another's influence, so that the majority decision is determined entirely by the behavior of independent voters, no matter how small this group is. In a large election, a majority of these almost surely identify the true state of the world. If there are no partisans then pure marginal voting delivers the same outcome in the limit: as participation is limited to increasingly elite voters, the election outcome tends toward unanimity, and the policy outcome converges to the desired extreme. A positive mass of partisan votes for either side, however, bounds the policy outcome away from 0 and 1, implying some utility loss, which is increasing in p. If the domain of F were bounded such that even the maximum level of expertise were some $q_{\max} < 1$ then pure marginal voting would fall short of pure pivotal voting even for the case of no partisans, because independent voters would be unable to coordinate unanimously on the desired side.

Another way of describing the result of Proposition 3 is to say that if votes had no marginal impact then voting would lead to *full information equivalence*, as Feddersen and Pesendorfer (1996, 1999) emphasize, meaning that large elections produce the same policy outcomes that would occur if information were made perfect. This no longer holds once the marginal impact of voting is taken into account, because full information equivalence requires that citizens unanimously rally around the correct extreme, which is impossible unless there are no partisans *and* the best informed independent voters are infallible. In less idyllic settings, the damage caused by mistaken votes and partisan votes is irreversible.[22]

[22] The result that welfare is higher in the absence of marginal motives is partly a consequence of the assumption that the optimal policy lies at one of the extremes of the policy space: if interior policies could also be optimal then there may be informational benefits to compromise. Thus, a tractable model in which the optimal policy can lie in the interior of the policy interval would be a useful direction for future extension. In addition to altering welfare implications, this might mitigate the marginal voter's curse: as noted above, a binary state gives citizens a strong reason to abstain, so as to be as nearly unanimous as possible. An interior optimum may also balance the two parties' vote shares such that, as in Myatt (2012), pivotal voting incentives shrink to zero at the same rate (i.e., $\frac{1}{n}$) as marginal voting incentives, rather than exponentially. If so, both motivations will remain relevant in large elections.

7 General Model

If θ is strictly between 0 and 1 then the electoral rule is a hybrid of the marginal voting and pivotal voting benchmarks, as given by equation (1). In that case, the expected benefit of voting is merely the weighted average of the expected benefits derived in Sections 4 and 5.

$$\Delta Eu(q) = \frac{1+q}{2}\left[\theta\left(\Delta\lambda_{+}\right) + (1-\theta)\Pr\left(\mathcal{P}_{+}\right)\right] - \frac{1-q}{2}\left[\theta\left(\Delta\lambda_{-}\right) + (1-\theta)\Pr\left(\mathcal{P}_{-}\right)\right]$$

This difference is positive if and only if q exceeds the following threshold.

$$\bar{q}_G^{br} = \frac{\theta\left(\Delta\lambda_{-} - \Delta\lambda_{+}\right) + (1-\theta)\left[\Pr\left(\mathcal{P}_{-}\right) - \Pr\left(\mathcal{P}_{+}\right)\right]}{\theta\left(\Delta\lambda_{+} + \Delta\lambda_{-}\right) + (1-\theta)\left[\Pr\left(\mathcal{P}_{+}\right) + \Pr\left(\mathcal{P}_{-}\right)\right]} \tag{31}$$

For any p, this threshold lies strictly between (12) and (26). Since $\bar{q}_P^{br}$ and $\bar{q}_M^{br}$ are both positive for any strategy in which a positive fraction of the electorate votes, (31) is positive as well, implying that the best response for citizens with the lowest levels of expertise is to abstain. Theorems 1 and 2 make no claim of equilibrium uniqueness, but if the distribution of expertise is such that there is a unique equilibrium participation threshold for each of these extreme electoral rules then the fact that (31) is between (12) and (26) implies that any equilibrium threshold $\bar{q}_G^*$ in the general model must be strictly between $\bar{q}_P^*$ and $\bar{q}_M^*$. As in the special cases of pure marginal or pure pivotal voting, the logic of McLennan (1998) implies that equilibrium abstention is good for social welfare.

Theorem 4 *If $0 < \theta < 1$ then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a threshold strategy $\sigma_{\bar{q}^*}$ with $\bar{q}_G^* > 0$. Moreover, such an equilibrium exists.*

The fact that $q_G^*(n)$ lies between $\bar{q}_P^*(n)$ and $\bar{q}_M^*(n)$ implies that the limit q^G of a sequence of equilibrium thresholds for a general model lies weakly between q^P and q^M. As noted above, however, the distance between these thresholds may be rather large. The goal of the rest of the section is to generate more specific predictions about the location of q^G, relative to the other two thresholds.

Equation (31) gives the equilibrium condition for a general model and a finite n. This equation depends on the marginal changes $\Delta\lambda_+$ and $\Delta\lambda_-$ in policy associated with additional votes for the superior and inferior parties, respectively, and the probabilities $\Pr(\mathcal{P}_+)$ and $\Pr(\mathcal{P}_-)$ of such votes being pivotal. The former can be rewritten as (13) and (14), which each have one term that decreases linearly in n and another that decreases exponentially with n, and the latter can be written as (27) and (28), which decrease exponentially with n. (31) is therefore equivalent to

$$\frac{1+\bar{q}}{1-\bar{q}} = \frac{\theta\left[\frac{v_+}{nv_\tau^2} + \frac{n\left(v_-^2 - v_+^2\right) - 2v_+}{2nv_\tau^2} e^{-nv_\tau}\right] + (1-\theta)\left[\frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+v_-}}} \frac{\sqrt{v_+} + \sqrt{v_-}}{\sqrt{v_-}}\right]}{\theta\left[\frac{v_-}{nv_\tau^2} + \frac{n\left(v_+^2 - v_-^2\right) - 2v_-}{2nv_\tau^2} e^{-nv_\tau}\right] + (1-\theta)\left[\frac{e^{-n\left(\sqrt{v_+} - \sqrt{v_-}\right)^2}}{4\sqrt{n\pi\sqrt{v_+v_-}}} \frac{\sqrt{v_+} + \sqrt{v_-}}{\sqrt{v_+}}\right]}$$

which reduces simply to

$$\frac{1+\bar{q}}{1-\bar{q}} = \frac{v_+}{v_-} = \rho(\bar{q}) \tag{32}$$

because exponential terms vanish more quickly than linear terms.[23]

The limit of any sequence of equilibrium thresholds $q_G^*(n)$ must be a solution to (32). By the arguments of Section 4, such a solution q^G exists and is unique. In fact, as Theorem 5 now states, it coincides exactly with q^M, because (32) is identical to the limiting equilibrium condition (16) for pure pure marginal voting. In other words, the analysis of pure pivotal voting is knife-edge: if the impact of a vote extends beyond its mechanical ability to change the identity of the election winner, even if its marginal impact is minimal, then equilibrium behavior in large elections differs dramatically from the pure pivotal voting case, and in fact is virtually identical to the opposite benchmark, in which there is no discontinuity at all at the 50% threshold.

[23] Castanheira (2003) and Faravelli, Man, and Walsh (2015) argue that this slower convergence rate of marginal voting can also help explain turnout in costly voting environments. In private value settings, pivot probabilities decrease exponentially only when the electoral split is uneven, and is exactly known: with a small amount of uncertainty regarding the exact electoral split, pivot probabilites can decrease at a linear rate instead (see Myatt 2012). The key ingredient for that result, however, is that either side might actually have the electoral advantage. Here, small uncertainty regarding the distribution of expertise or other parameters of the game would not change the result that pivot probabilities decrease exponentially with n, because every citizen with $q_i > 0$ is more likely to vote for the superior party. The average level of expertise $E(q_i|i \text{ votes})$ might not be known precisely, but is unambiguously positive.

Theorem 5 *If $\theta \in (0,1]$, then $q^G = q^M$.*

The general model analyzed in this section has only a single discontinuity, at the 50% threshold. In some applications, it might be reasonable to assume additional discontinuities. In the U.S. Senate, for example, a party that obtains at least 2/3 of the power can prevent the opposing party from using filibusters to block legislative actions. In situations like this, it might be reasonable to assume that there is a large discontinuity at the 50% threshold and smaller discontinuities at the $\frac{1}{3}$ and $\frac{2}{3}$ thresholds. The logic that pivotal events become exponentially less important than marginal considerations, however, relies in no way on there being a unique pivotal event; with two or more pivotal events, each should become exponentially less likely as n grows large, so that in the limit, any marginal changes away from the pivotal events will be all-important, and the combined set of pivotal discontinuities will be irrelevant.

8 Nonlinear Policy Functions

The model of Section 3 assumes that the policy outcome $x(a,b) = \frac{a}{a+b} = \lambda_+$ in the marginal voting component of the general model is simply equal to the vote share of the superior party. This section considers a more general *policy function* $\psi : \mathbb{Z}_+^2 \to [0,1]$ and shows that the marginal voter's curse still arises as long as $x = \psi(a,b)$ satisfies the following three conditions.

Condition 1 (Monotonicity) *$\psi(a,b)$ increases in a and decreases in b.*

Condition 2 (Symmetry) *For any $a, b \in \mathbb{Z}_+$, $\psi(b,a) = 1 - \psi(a,b)$.*

Condition 3 (Underdog) *$|\psi(a,b+1) - \psi(a,b)| - |\psi(a+1,b) - \psi(a,b)|$ has the same sign as $a - b$.*

The monotonicity condition merely states that A and B votes push the policy outcome toward 1 and 0 and therefore increase and decrease utility, respectively. Symmetry implies that reversing the numbers of votes that each party receives exactly reverses the parties' power, for example implying that $\psi(a,b) = \frac{1}{2}$ when $a = b$. The underdog property states that the impact of one additional vote for the party that has fewer votes is greater than the impact of one additional vote for the party with a majority. If ψ is a monotonic and symmetric function of the vote share $\lambda_+ = \frac{a}{a+b}$ then Condition 3 is implied if ψ is *S-shaped*—that is, convex for vote shares in $\left[0, \frac{1}{2}\right]$ and concave for vote shares in $\left[\frac{1}{2}, 1\right]$, meaning that the vote share has a diminishing marginal impact on the majority party's power. Other examples of policy functions that satisfy Conditions 1 through 3 include any "contest" function of the form

$$\psi(a,b) = \frac{a^z}{a^z + b^z}$$

with positive z. Such contest functions are S-shaped for $z > 1$ but an inverted S-shape (i.e. concave and then convex) for $z < 1$. Theorem 6 now generalizes Theorem 4 to state that a marginal voter's curse arises for any policy function that satisfies Conditions 1 through 3.

Theorem 6 *If $\psi : \mathbb{Z}_+^2 \to [0,1]$ satisfies Conditions 1 through 3 then $\sigma^* \in \Sigma$ is a Bayesian Nash equilibrium only if it is a threshold strategy $\sigma_{\bar{q}^*}$ with $\bar{q}^* > 0$. Moreover, such an equilibrium exists.*

Condition 3 highlights an important similarity between the swing voter's curse and the marginal voter's curse, which is that these curses arise from two manifestations of the same phenomenon, namely that an electoral rule exhibits an underdog property. In pivotal voting environments, a vote for the losing party is more likely to be pivotal than a vote for the winning party; in marginal voting environments, a vote for the losing party induces a larger policy shift than a vote for the winning party. Ex ante, parties in this model are symmetric: there is no winning party and no losing party. When citizens vote informatively, however, the party with the superior policy platform receives more votes (in expectation) than the inferior party. Thus, a common-value environment translates either manifestation of the underdog property into an incentive for poorly informed citizens to abstain from voting, in deference to those with better expertise.

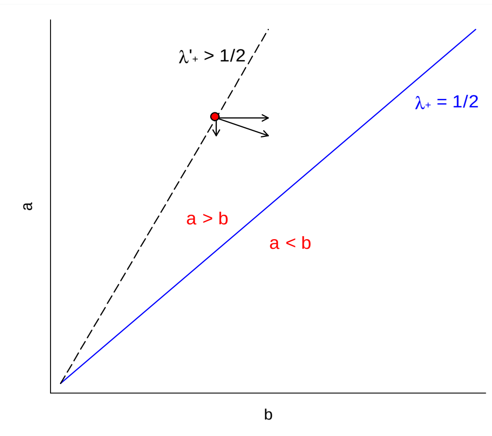

Figure 6: Isolevels of $\psi(a,b)$

As noted above, the underdog property—which is so central to the logic of the marginal voter's curse—is satisfied by a large class of continuous policy functions. Asymptotically, the condition actually holds for *any* increasing and differentiable function $\psi(\lambda_+)$ of the vote share $\lambda_+ = \frac{a}{a+b}$. To see this, note that, given monotonicity, the difference $|\psi(a,b+1) - \psi(a,b)| - |\psi(a+1,b) - \psi(a,b)|$ can be rewritten as follows.

$$-[\psi(a,b+1) - \psi(a,b)] - [(a+1,b) - \psi(a,b)]$$

As a and b grow large (keeping the vote share λ_+ fixed), this converges to the following,

$$-\frac{\partial}{\partial b}\psi(a,b) - \frac{\partial}{\partial a}\psi(a,b) = -\psi'(\lambda_+)\frac{-a}{(a+b)^2} - \psi'(\lambda_+)\frac{b}{(a+b)^2}$$

which has the same sign as $a-b$.

An intuition for the asymptotic genericity of Condition 3 can be aided by Figure 6, which plots isolevels of $\psi(a,b)$ for vote totals a and b. When ψ depends on a and b only through the vote share λ_+, sets of vote totals that generate the same policy outcome, the isolevels, correspond to rays from the origin. For example, the 45-degree line corresponds to exact ties, or a vote share of $\lambda_+ = \frac{1}{2}$. The steeper dashed line corresponds to a higher vote share $\lambda'_+ > \frac{1}{2}$.For large a and b, the vector of changes to the policy outcome generated by increases in a and b is proportional to the gradient, which by definition is perpendicular to the isolevel. In the region where $a > b$, therefore, the policy reaction to a B vote exceeds the reaction to an A vote, while the opposite is true in the region where $a < b$.

9 Conclusion

That voters should focus on the rare event of a pivotal vote is often viewed as the central hallmark of rationality in models of elections. In showing the important consequences that this has for voters in common-value environments, Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996) have inspired a revival of the classic common-value paradigm of Condorcet (1785), and subsequent literature has continued to document ways in which the pivotal voting calculus dramatically shapes voting behavior. This paper has embraced the common-value paradigm, but constructed a model in light of recent evidence that margins of victory matter even away from the 50% threshold. This has led to the discovery of a new strategic incentive for voter abstention, as well as the important realization that marginal voting incentives dominate pivotal voting considerations. While perhaps intuitive, the rationale behind this verdict is quantitatively based, relying on the comparison of asymptotic orders of magnitude of effects in large elections. The implication is that the predictions of a purely pivotal voting model are knife-edge in a more general setting.

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Appendix

Proof of Theorem 1. As explained in the text, σ is an equilibrium only if it is a quality threshold strategy, with threshold given by (12). Conditional on the total number $\tau = a + b$ of votes, the precise numbers of votes a and b for either side follow binomial distributions, with probability parameters $\frac{v_+}{v_\tau}$ and $\frac{v_-}{v_\tau}$. As long as not all citizens abstain, (4) exceeds (5), implying that the distribution of the number a of correct votes first-order stochastically dominates the distribution of the number b of incorrect votes. Keeping the total $a + b$ fixed, however, $\frac{a+1}{a+b+1} - \frac{a}{a+b}$ decreases in a and increases in b, implying that the distribution of $\Delta\lambda_-$ first-order stochastically dominates the distribution of $\Delta\lambda_+$. In particular, then, $E_{a,b}(\Delta\lambda_- | a + b = \tau) > E_{a,b}(\Delta\lambda_+ | a + b = \tau)$ and, taking expectations over τ, $E_{a,b}(\Delta\lambda_-) > E_{a,b}(\Delta\lambda_+)$. This implies that (12) is positive.

Equilibrium existence follows because the best response to a threshold strategy $\sigma_{\bar{q}}$ is another $\sigma_{\bar{q}_M^{br}}$. Since the best-response threshold $\bar{q}_M^{br}$ depends continuously on $\bar{q}$ and the set $[0, 1]$ of possible thresholds is compact, a fixed point $\bar{q}^* = \bar{q}_M^{br}(\bar{q}^*)$ exists by Brouwer's theorem, that characterizes a voting strategy $\sigma_{\bar{q}^*}$ that is its own best response, and therefore a Bayesian Nash equilibrium of the game. ■

Proof of Lemma 1. The expected vote share of the superior party can be written as

$$\begin{aligned} E_{a,b}[\lambda_+(a,b)] &= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty} e^{-n_\tau}\frac{n_+^a}{a!}\frac{n_-^b}{b!}u[x(a,b)] \\ &= e^{-n_\tau}\sum_{a=1}^{\infty}\sum_{b=0}^{\infty}\frac{n_+^a}{a!}\frac{n_-^b}{b!}\left(\frac{a}{a+b}\right) + \frac{1}{2}e^{-n_\tau} \\ &= e^{-n_\tau}\sum_{a=1}^{\infty}\left[\frac{1}{(a-1)!}\frac{n_+^a}{n_-^a}\sum_{b=0}^{\infty}\frac{n_-^{a+b}}{b!(a+b)}\right] + \frac{1}{2}e^{-n_\tau} \end{aligned}$$

where the second equality follows because $x(0,0) = \frac{1}{2}$. Differentiating and integrating the innermost summand as follows,

$$\begin{aligned} \sum_{b=0}^{\infty}\frac{n_-^{a+b}}{b!(a+b)} &= \sum_{b=0}^{\infty}\int_0^{n_-}\frac{d}{dt}\left(\frac{t^{a+b}}{b!}\frac{1}{a+b}\right)dt \\ &= \int_0^{n_-}\sum_{b=0}^{\infty}\left(\frac{t^{a+b-1}}{b!}\right)dt \\ &= \int_0^{n_-} t^{a-1}e^t dt, \end{aligned}$$

this reduces further to the following.

$$\begin{aligned}
E_{a,b}\left[\lambda_+ (a,b)\right] &= e^{-n_\tau}\frac{n_+}{n_-}\int_0^{n_-}\sum_{a=1}^{\infty}\frac{\left(\frac{n_+}{n_-}t\right)^{a-1}}{(a-1)!}e^t dt + \frac{1}{2}e^{-n_\tau} \\
&= e^{-n_\tau}\frac{n_+}{n_-}\int_0^{n_-} e^{\left(\frac{n_+}{n_-}t\right)}e^t dt + \frac{1}{2}e^{-n_\tau} \\
&= e^{-n_\tau}\frac{n_+}{n_-}\int_0^{n_-} e^{\left(\frac{n_\tau}{n_-}\right)t} dt + \frac{1}{2}e^{-n_\tau} \\
&= e^{-n_\tau}\frac{n_+}{n_\tau}\left(e^{n_\tau}-1\right) + \frac{1}{2}e^{-n_\tau} \\
&= \frac{n_+}{n_\tau} + \frac{n_- - n_+}{2n_\tau}e^{-n_\tau} \qquad (33)
\end{aligned}$$

If a citizen votes for the party with the superior platform, this increases the expected vote share to

$$\begin{aligned}
E_{a,b}\left[\lambda_+ (a+1,b)\right] &= \sum_{a=0}^{\infty}\sum_{b=0}^{\infty} e^{-n_\tau}\left(\frac{n_+^a}{a!}\right)\left(\frac{n_-^b}{b!}\right)\frac{a+1}{a+b+1} \\
&= e^{-n_\tau}\sum_{a=0}^{\infty}\left[\frac{a+1}{a!}\frac{n_+^a}{n_-^{a+1}}\sum_{b=0}^{\infty}\frac{n_-^{a+b+1}}{b!\,(a+b+1)}\right]
\end{aligned}$$

which, differentiating and integrating as before, reduces further as follows.

$$\begin{aligned}
&= e^{-n_\tau}\sum_{a=0}^{\infty}\frac{a+1}{a!}\frac{n_+^a}{n_-^{a+1}}\int_0^{n_-} t^a e^t dt \\
&= e^{-n_\tau}\int_0^{n_-}\sum_{a=0}^{\infty}\left(\frac{a}{a!}\frac{n_+^a}{n_-^{a+1}}t^a + \frac{1}{a!}\frac{n_+^a}{n_-^{a+1}}t^a\right)e^t dt. \\
&= e^{-n_\tau}\int_0^{n_-}\left[\frac{n_+}{n_-^2}t\sum_{a=1}^{\infty}\frac{\left(\frac{n_+}{n_-}t\right)^{a-1}}{(a-1)!} + \frac{1}{n_-}\sum_{a=0}^{\infty}\frac{\left(\frac{n_+}{n_-}t\right)^{a}}{a!}\right]e^t dt \\
&= e^{-n_\tau}\int_0^{n_-}\left[\frac{n_+}{n_-^2}te^{\left(\frac{n_+}{n_-}t\right)} + \frac{1}{n_-}e^{\left(\frac{n_+}{n_-}t\right)}\right]e^t dt \\
&= \frac{n_+}{n_-^2}e^{-n_\tau}\int_0^{n_-} te^{\left(\frac{n_\tau}{n_-}\right)t}dt + \frac{1}{n_-}e^{-n_\tau}\int_0^{n_-} e^{\left(\frac{n_\tau}{n_-}\right)t}dt
\end{aligned}$$

Integrating by parts, this reduces to the following.

$$\begin{aligned}
& \frac{n_+}{n_-^2}e^{-n_\tau}\left[\frac{n_-^2 e^{n_\tau}}{n_\tau} - 0 - \frac{n_-}{n_\tau}\int_0^{n_-} e^{\left(\frac{n_\tau}{n_-}\right)t}\right] + \frac{1}{n_\tau}e^{-n_\tau}\left(e^{n_\tau}-1\right) \\
= & \frac{n_+}{n_-^2}e^{-n_\tau}\left[\frac{n_-^2 e^{n_\tau}}{n_\tau} - \left(\frac{n_-}{n_\tau}\right)^2\left(e^{n_\tau}-1\right)\right] + \frac{1}{n_\tau}\left(1-e^{-n_\tau}\right) \\
= & \left(\frac{n_+}{n_\tau} - \frac{n_+}{n_\tau^2} + \frac{1}{n_\tau}\right) + \left(\frac{n_+}{n_\tau^2} - \frac{1}{n_\tau}\right)e^{-n_\tau} \\
= & \frac{n_+\left(n_\tau\right)+n_-}{n_\tau^2} - \frac{n_-}{n_\tau^2}e^{-n_\tau} && (34)
\end{aligned}$$

The difference $E_{a,b}\left(\Delta\lambda_+\right)$ between (33) and (34) is then given by

$$E_{a,b}\left(\Delta\lambda_+\right) = \frac{n_-}{n_\tau^2} + \frac{n_+^2 - n_-^2 - 2n_-}{2n_\tau^2}e^{-nv_\tau}$$

which is equivalent to (13). The difference $E_{a,b}\left(\Delta\lambda_-\right)$ is given by (14), by an analogous derivation. ■

Proof of Proposition 1. The left- and right-hand sides of the equilibrium condition (15) are continuous in n and in $\bar{q}$ (through the continuous functions v_+, v_-, and v_τ) so a sequence $\bar{q}_M^*\left(n\right)$ of solutions to (15) must converge to a solution of the limit equilibrium condition (15). If $\lim_{n\to\infty} v_\tau\left[\bar{q}_M^*\left(n\right)\right]$ is positive then the limit of (15) is given by (16), because the terms in parentheses in both the numerator and denominator of the right-hand side of (15) are bounded in absolute value by $n+2$. The limit of e^{-nv_τ} can be positive only if $\lim_{n\to\infty} v_\tau = 0$, implying that v_+ and v_- both converge to zero. But (15) converges to (16) in that case, as well.

As $\bar{q}$ increases from zero to one, the right-hand side of (16) increases from one to infinity. For any p, differentiating (18) and (19) with respect to $\bar{q}$ yields

$$\begin{aligned}
v_+' &= -\frac{I}{2}\left(1+\bar{q}\right)f\left(\bar{q}\right) \\
v_-' &= -\frac{I}{2}\left(1-\bar{q}\right)f\left(\bar{q}\right)
\end{aligned}$$

and differentiating (20) therefore yields

$$\rho'\left(\bar{q}\right) = \frac{v_+'v_- - v_+v_-'}{v_-^2} = \frac{I}{2}f\left(\bar{q}\right)\frac{-\left(1+\bar{q}\right)v_- + \left(1-\bar{q}\right)v_+}{v_-^2}$$

which is positive if and only if the left-hand side of (16) exceeds the right-hand side. In other words, any solution to (16) also constitutes the unique maximum of $\rho\left(\bar{q}\right)$; clearly, there is exactly one such solution.

If $p=0$ then (20) reduces to $\rho(\bar{q})=\frac{1+m(\bar{q})}{1-m(\bar{q})}$ which is increasing for all $\bar{q}$, implying that the limit of equilibrium behavior is the corner solution $q^M=1$. If $p>0$ then $\rho(0)>1$ and $\rho(1)=1$, so the solution q^M is strictly between 0 and 1, as claimed in (i). (20) also decreases in K (and therefore in p) for all $\bar{q}$, implying that the solution q^M to (16) strictly decreases in K (and therefore in p), as claimed in (ii).

To show claim (iii), first rewrite (20) as

$$\rho(\bar{q})=\frac{K+\int_{\bar{q}}^{1}(1+q)\,f(q)\,dq}{K+\int_{\bar{q}}^{1}(1-q)\,f(q)\,dq}=\frac{\int_0^1\Gamma_+(q)\,f(q)\,dq}{\int_0^1\Gamma_-(q)\,f(q)\,dq}$$

in terms of $\Gamma_+(q)=K+I_{\bar{q}}(1+q)$ and $\Gamma_-(q)=K+I_{\bar{q}}(1-q)$, where $I_{\bar{q}}$ is the indicator function of the interval $[\bar{q},1]$. For any K, $\Gamma_+(q)$ and $\Gamma_-(q)$ are non-negative and respectively non-decreasing and non-increasing functions of q.

With this notation, consider a distribution g such that $\frac{g(q)}{f(q)}$ is increasing, thus satisfying the monotone likelihood ratio property MLRP. For any K and any $\bar{q}\in[0,1)$, we will show that $\rho(\bar{q})$ is higher under g than under f, implying that the solution q^M to (16) is lower under g than under f. That is, we will show the following,

$$\frac{\int_0^1\Gamma_+(q)\,g(q)\,dq}{\int_0^1\Gamma_-(q)\,g(q)\,dq}>\frac{\int_0^1\Gamma_+(q)\,f(q)\,dq}{\int_0^1\Gamma_-(q)\,f(q)\,dq}$$

which is equivalent to the following.

$$\int_0^1\int_0^1\Gamma_+(q)\,\Gamma_-\left(q'\right)f\left(q'\right)g(q)\,dqdq'>\int_0^1\int_0^1\Gamma_+(q)\,\Gamma_-\left(q'\right)f(q)\,g\left(q'\right)dqdq'$$

Equivalently, we will show that the expression

$$\Gamma=\int_0^1\int_0^1\Gamma_+(q)\,\Gamma_-\left(q'\right)\left[f(q)\,g\left(q'\right)-f\left(q'\right)g(q)\right]dqdq' \tag{35}$$

is positive.

Splitting the domain of integration into two symmetric parts produces the following.

$$\begin{aligned}\Gamma &= \iint\limits_{q>\tilde{q}}\Gamma_+(q)\,\Gamma_-(\tilde{q})\left[f(\tilde{q})\,g(q)-f(q)\,g(\tilde{q})\right]dqd\tilde{q}\\ &\quad+\iint\limits_{\tilde{q}>q}\Gamma_+(q)\,\Gamma_-(\tilde{q})\left[f(\tilde{q})\,g(q)-f(q)\,g(\tilde{q})\right]dqd\tilde{q}\end{aligned}$$

Reversing the labels of q and $\tilde{q}$ in the second double integral and collecting similar terms, this is equivalent to

$$\begin{aligned}\Gamma &= \iint\limits_{q>\tilde{q}} \left[\Gamma_+(q)\Gamma_-(\tilde{q}) - \Gamma_+(\tilde{q})\Gamma_-(q)\right]\left[f(\tilde{q})g(q) - f(q)g(\tilde{q})\right] dqd\tilde{q} \\ &= \iint\limits_{q>\tilde{q}} \Gamma_-(q)\Gamma_-(\tilde{q})\left[\frac{\Gamma_+(q)}{\Gamma_-(q)} - \frac{\Gamma_+(\tilde{q})}{\Gamma_-(\tilde{q})}\right] f(q) f(\tilde{q}) \left[\frac{g(q)}{f(q)} - \frac{g(\tilde{q})}{f(\tilde{q})}\right] dqd\tilde{q}\end{aligned}$$

This expression is positive, as desired, because Γ_+ and $\frac{g(q)}{f(q)}$ are non-decreasing in q and Γ_- is non-increasing in q. ■

Proof of Theorem 2. As argued in the text, σ is an equilibrium only if it is a quality threshold strategy, with threshold given by (26). Conditional on the total number $a+b$ of votes, the precise numbers of votes a and b for either side follow binomial distributions, with probability parameters $\frac{v_+}{v_\tau}$ and $\frac{v_-}{v_\tau}$. As long as not all citizens abstain, (4) exceeds (5), implying from (6) that $\Pr(a=k+1, b=k)$ exceeds $\Pr(a=k, b=k+1)$ for any k and, taking averages over k, that $\Pr(a=b+1)$ exceeds $\Pr(a=b)$. (24) therefore exceeds (23), implying that (26) is positive.

Since the best response to a threshold strategy $\sigma_{\bar{q}}$ is another threshold strategy $\sigma_{\bar{q}_P^{br}}$, where the best-response threshold $\bar{q}_P^{br}$ depends continuously on $\bar{q}$ and the set $[0,1]$ of possible thresholds is compact, a fixed point $\bar{q}^* = \bar{q}_P^{br}(\bar{q}^*)$ exists by Brouwer's theorem, that characterizes a voting strategy $\sigma_{\bar{q}^*}$ that is its own best response, and therefore a Bayesian Nash equilibrium of the game. ■

Proof of Proposition 2. The left-hand side of (30) approaches infinity as $\bar{q}$ approaches 0 and approaches zero as $\bar{q}$ approaches 1. $\bar{q}=1$ is therefore a solution to (30) if and only if $K=0$. In that case, however, Theorem 3 of McMurray (2013) implies that no sequence of equilibrium thresholds can converge to one, because the assumption that f is log-concave implies that $\lim_{q\to 1}\frac{f'(q)}{f(q)} < \infty$.[24] Moreover, Theorem 4 of McMurray (2013) implies that there is a unique solution q_0^P to (30) in the open interval $(0,1)$. This implies that the left-hand side of (30) is positive on the interval $\left[0, q_0^P\right)$ and negative on the interval $\left(q_0^P, 1\right)$.

When the left-hand side of (30) is positive, it is strictly decreasing in $\bar{q}$. To see this, differentiate the left-hand side of (30) to obtain the following.

$$-f(\bar{q})\left(\frac{1+\bar{q}^2}{2}\frac{m(\bar{q})}{\bar{q}} - 1\right) + \frac{1-F(\bar{q})}{2}\left(-\frac{1+q^2}{q^2}m(\bar{q}) + \frac{1+q^2}{q}m'(\bar{q})\right) \tag{36}$$

[24]As that paper notes, this condition merely rules out electorates that are arbitrarily close to being perfectly informed, and is sufficient for the result but not necessary.

Since f is assumed to be log-concave, Lemma 2 of Bagnoli and Bergstrom (2005) states that $E(q-\bar{q}|q \geq \bar{q})$ decreases with $\bar{q}$ or, equivalently, that $m'(\bar{q}) < 1$, implying that the second term in (36) is negative. If the left-hand side of (30) is positive then the first term of (36) is negative, as well.

That the left-hand side of (30) is decreasing whenever it is positive implies a unique solution q^P to (30) for any partisanship ratio K, thus establishing (i). This also makes clear that if K increases (or, equivalently, if p increases) then q^P decreases, thus establishing (ii). The proof of (iii) is then analogous to the proof of part (iii) of Proposition 1. ■

Proof of Theorem 3. As the proofs of Propositions 1 and 2 show, $\rho(\bar{q})$ exceeds the right-hand side of (16) if and only if $\bar{q} < q^M$ and exceeds the right-hand side of (29) if and only if $\bar{q} < q^P$. The right-hand side of (29) exceeds the right-hand side of (16) for any $\bar{q} > 0$ (and when $\bar{q} = 0$ both equal one, whereas $\rho(0) > 1$), however, so in particular $\rho(q^P)$ exceeds the right-hand side of (16), implying that $q^P < q^M$. ■

Proof of Proposition 3. As n grows large, equilibrium strategies converge pointwise to σ_{q^P} under pure pivotal voting and to σ_{q^M} under pure marginal voting. In either regime, actual vote shares converge in probability to expected vote shares. That is, $\frac{a}{b} \to_p \frac{nv_+}{nv_-} = \frac{v_+}{v_-} = \rho$. Under pure pivotal voting, expected utility is given by $\Pr\left(\frac{a}{b} > 1\right)$, which therefore converges to $u^P = 1$ since $\frac{a}{b}$ converges to $\rho\left(q^P\right) > 1$. This logic is valid for any $p < 1$.[25] Under pure marginal voting, utility is given by $\frac{a}{a+b}$, which converges in probability to $u^M = \frac{nv_+}{nv_+ + nv_-} = \frac{\rho(q^M)}{\rho(q^M)+1}$. This decreases in p since it increases in $\rho\left(q^M\right)$, which decreases in p (as shown in Section 4). As that section explains, $p = 0$ implies that $\lim_{\bar{q}\to q^M} \rho(\bar{q}) = \infty$, therefore implying that $u^M = 1$. If $p = 1$ then from (18) and (19) it is also clear that $\rho(\bar{q}) = 1$ for any $\bar{q}$, implying that $u^M = \frac{1}{2}$ in that case. ■

Proof of Theorem 4. That a Bayesian Nash equilibrium must be a threshold strategy $\sigma_{\bar{q}^*}$ with $\bar{q}^*_G > 0$ is explained in the text. Equilibrium existence follows because the best response to a threshold strategy $\sigma_{\bar{q}}$ is another $\sigma_{\bar{q}^{br}_G}$. Since the best-response threshold $\bar{q}^{br}_G$ depends continuously on $\bar{q}$ and the set $[0,1]$ of possible thresholds is compact, a fixed point $\bar{q}^*_G = \bar{q}^{br}_G(\bar{q}^*)$ exists by Brouwer's theorem, that characterizes a voting strategy $\sigma_{\bar{q}^*}$ that is its own best response, and therefore a Bayesian Nash equilibrium of the game. ■

Proof of Theorem 6. With a generalized policy function, the expected benefit of voting (9) can be rewritten as follows,

$$\Delta Eu(q) = E_{a,b}\left[\frac{1}{2}(1+q)\Delta_+\psi(a,b) + \frac{1}{2}(1-q)\Delta_- x(a,b)\right]$$

in terms of the positive difference in policy $\Delta_+\psi(a,b) = \psi(a+1,b) - \psi(a,b)$ induced by

[25] For $p = 1$, $\Pr(n_\alpha > n_\beta) = \Pr(n_\alpha < n_\beta)$ for any n, implying that $u^M = \frac{1}{2} + 0$.

one additional vote for the superior party A and the negative policy difference $\Delta_{-}\psi(a,b) = \psi(a,b+1) - \psi(a,b)$ induced by one additional vote for party B. Given the symmetry condition, the latter difference can be written as

$$\begin{aligned}\Delta_{-}\psi(a,b) &= [1-\psi(b+1,a)] - [1-\psi(b,a)] \\ &= \psi(b,a) - \psi(b+1,a) \\ &= -\Delta_{+}\psi(b,a)\end{aligned}$$

in terms of the positive impact of a correct vote when there are b votes for party A and a votes for party B.

Since $\Delta_{+}\psi(a,b)$ and $\Delta_{+}\psi(b,a)$ are both positive, (9) is positive if and only if q exceeds the following threshold,

$$\bar{q}_{M}^{br} = \frac{E_{a,b}[\Delta_{+}\psi(b,a)] - E_{a,b}[\Delta_{+}\psi(a,b)]}{E_{a,b}[\Delta_{+}\psi(a,b)] + E_{a,b}[\Delta_{+}\psi(b,a)]}$$

which generalizes (12). The denominator of this expression is positive, and the numerator reduces as follows.

$$\begin{aligned}&\sum_{a,b}[\Delta_{+}\psi(b,a) - \Delta_{+}\psi(a,b)]\frac{e^{-n_{+}-n_{-}}n_{+}^{a}n_{-}^{b}}{a!b!} \\ =\; &\sum_{a>b}[\Delta_{+}\psi(b,a) - \Delta_{+}\psi(a,b)]\frac{e^{-n_{+}-n_{-}}n_{+}^{a}n_{-}^{b}}{a!b!} \\ &+\sum_{a<b}[\Delta_{+}\psi(b,a) - \Delta_{+}\psi(a,b)]\frac{e^{-n_{+}-n_{-}}n_{+}^{a}n_{-}^{b}}{a!b!}\end{aligned}$$

Relabeling variables in the second summation yields the following.

$$\begin{aligned}&\sum_{a>b}[\Delta_{+}\psi(b,a) - \Delta_{+}\psi(a,b)]\frac{e^{-n_{+}-n_{-}}n_{+}^{a}n_{-}^{b}}{a!b!} \\ &+\sum_{b<a}[\Delta_{+}\psi(a,b) - \Delta_{+}\psi(b,a)]\frac{e^{-n_{+}-n_{-}}n_{+}^{b}n_{-}^{a}}{a!b!} \\ =\; &\sum_{a>b}[\Delta_{+}\psi(b,a) - \Delta_{+}\psi(a,b)]\frac{e^{-n_{+}-n_{-}}n_{+}^{b}n_{-}^{b}}{a!b!}\left(n_{+}^{a-b} - n_{-}^{a-b}\right)\end{aligned}$$

Monotonicity and the underdog property together imply that $\Delta_{+}\psi(a,b) < \Delta_{+}\psi(b,a)$ if and only if $a > b$. This, together with the fact that $n_{+} > n_{-}$, implies that the above expression is strictly positive, and therefore that $\bar{q}_{M}^{br}$ is positive. That the best-response threshold is strictly positive implies that any equilibrium threshold is positive as well, and a positive fraction of the electorate therefore prefer to abstain in equilibrium. ■

Proof of Theorem 5. This theorem is proven in the text. ■